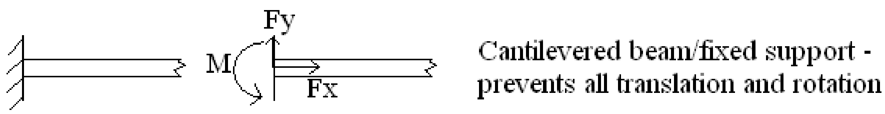
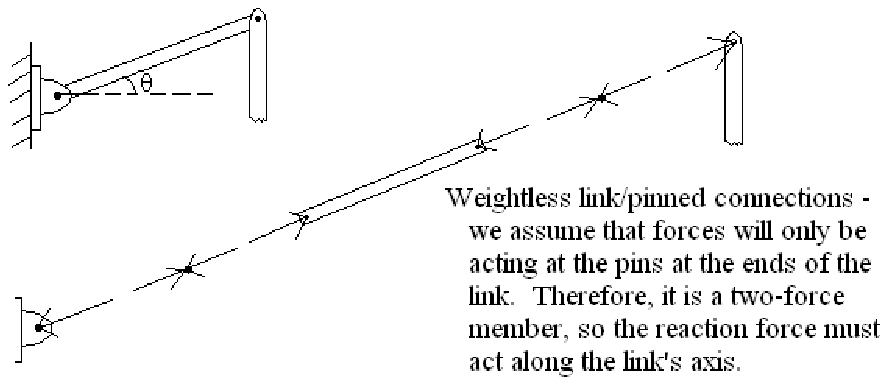
Trusses

Assumption: no friction, connections treated as points





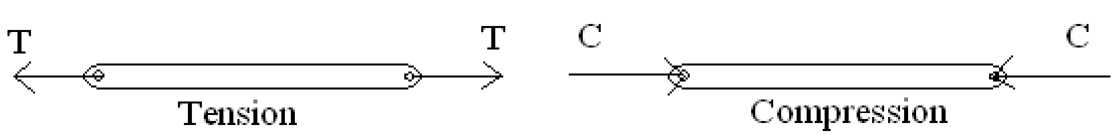
The weightless link will become important in the following section when we talk about trusses.

A truss is a structure composed of slender members joined together at their end points.

Truss assumptions:

1) All loadings are applied at the joints.
2) The members are joined together by smooth pins. In cases where bolted or welded joint connections are used, this assumption is satisfactory, provided the joining members intersect at a common point.

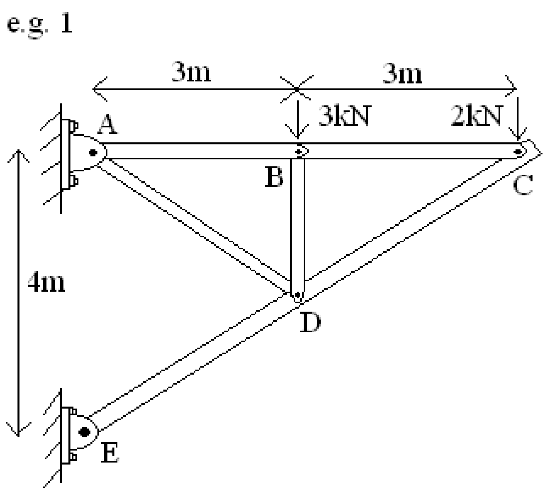
So, each truss member acts as a two-force member. Therefore, the forces at the ends of the member must be directed along the axis of the member.

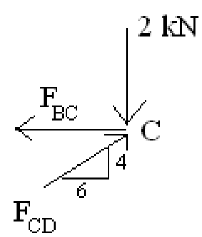


The Method of Joints

To find support reactions, we would normally ignore the forces within the members since they are internal to the support reaction's free body diagram. Instead, if we consider the equilibrium at each joint of the truss, then the member forces become external forces on the FBDs of the joints. The force system acting at each joint is concurrent. $\sum \text{Fx} = 0$ and $\sum \text{Fy} = 0$ must be satisfied for equilibrium. Therefore, we should start at a joint having at least one known and at most two unknown forces.
The correct sense of direction of an unknown member force (tensile or compressive) can in many cases be determined by "inspection." In more complicated cases, the sense of the unknown member force can be assumed. A positive answer indicates that you guessed correctly. A negative answer indicates that the sense shown on your FBD must be reversed. In this latter case, write the answer as positive but change (C) to (T) or vica-versa.

e.g. 1
Given: Dimensions of a truss for a balcony. Resultant loads act at B and C.
Find: The force in each member and state compression (C) or tension (T).

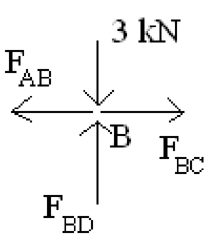




$$\xrightarrow{+} \sum Fx : F_{CD}\left(\frac{6}{\sqrt{6^2+4^2}}\right) - F_{BC} = 0$$

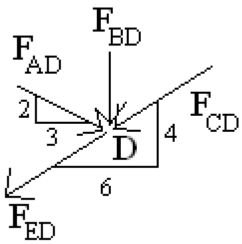
$$+\uparrow \sum Fy : F_{CD}\left(\frac{4}{\sqrt{6^2+4^2}}\right) - 2 = 0$$

F_{CD} = ***3.61kN (C)*** F_{BC} = ***3kN (T)***



$$\xrightarrow{+} \sum Fx : 3 - F_{AB} = 0$$ F_{AB} = ***3kN (T)***

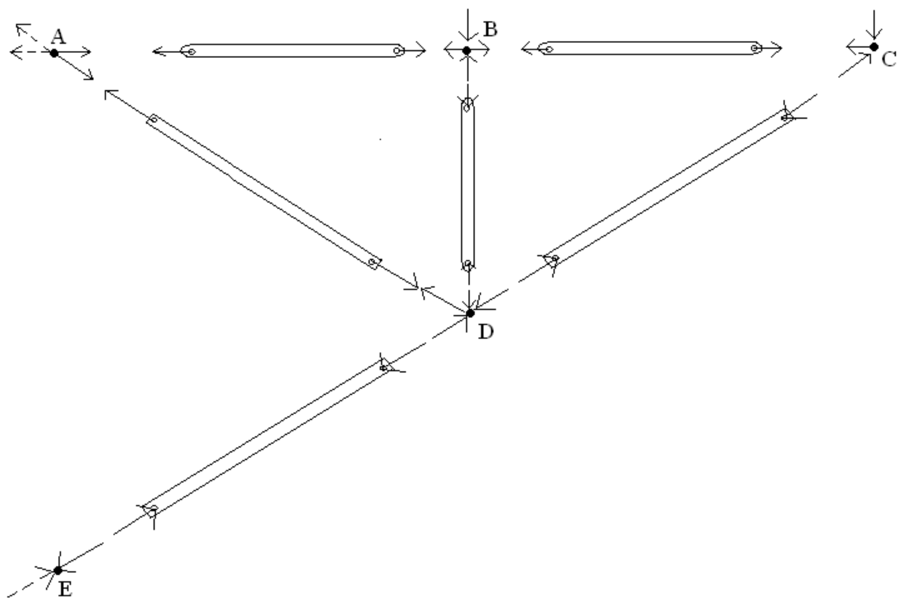
$$+\uparrow \sum Fy : F_{BD} - 3 = 0$$ F_{BD} = ***3kN (C)***



$$\xrightarrow[+]{} \sum Fx: F_{AD}\left(\frac{3}{\sqrt{3^2+2^2}}\right) - 3.61\left(\frac{6}{\sqrt{6^2+4^2}}\right) - F_{ED}\left(\frac{6}{\sqrt{6^2+4^2}}\right) = 0$$

$$+\uparrow \sum Fy: -3 - F_{AD}\left(\frac{2}{\sqrt{3^2+2^2}}\right) - 3.61\left(\frac{4}{\sqrt{6^2+4^2}}\right) - F_{ED}\left(\frac{4}{\sqrt{6^2+4^2}}\right) = 0$$

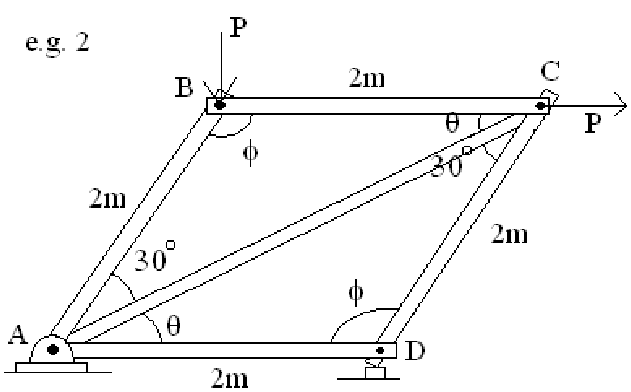
*F_{AD} = -2.7 $\Rightarrow$ F_{AD} = **2.7kN (T)*** *F_{ED} = -6.31 $\Rightarrow$ F_{ED} = **6.31kN (C)***

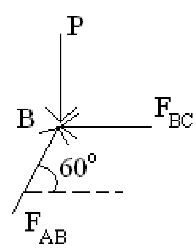


e.g. 2

Given: Dimensions of a truss (see pic below). T_{max} = 2kN, C_{max} = 1.2kN, for any member. Vertical external force P at B and horizontal force at C of equal magnitude P.

Find: Pmax





θ = 30° (law of sines)

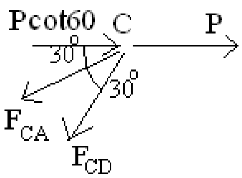
ϕ = 120°

$$\xrightarrow[+]{} \sum Fx: F_{AB}\cos 60° - F_{BC} = 0$$

$$+\uparrow \sum Fy: F_{AB}\sin 60° - P = 0$$

$$F_{AB} = \frac{P}{\sin 60°} = 1.155P \;\; (C)$$

$$F_{BC} = P\cot 60° = .577P \;\; (C)$$

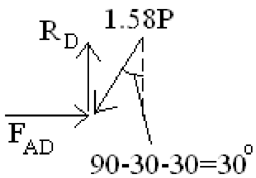


$$\xrightarrow[+]{} \sum Fx : P + P\cot 60° - F_{CA}\cos 30° - F_{CD}\cos 60° = 0$$

$$+\uparrow \sum Fy : -F_{CA}\sin 30° - F_{CD}\sin 60° = 0$$

$$F_{CA} = \frac{P + P\cot 60°}{\cos 30° - \sin 30°\cot 60°} = 2.73P \ \ (T)$$

$$F_{CD} = \frac{-2.73P\sin 30°}{\sin 60°} = -1.58P \Rightarrow F_{CD} = 1.58P \ \ (C)$$



$$\xrightarrow[+]{} \sum Fx : F_{AD} - 1.58P\sin 30° = 0$$

$$F_{AD} = 1.58\sin 30° P = .79P \ \ (C)$$

Tension$_{max}$ *= 2.73P*
2kN = 2.73P

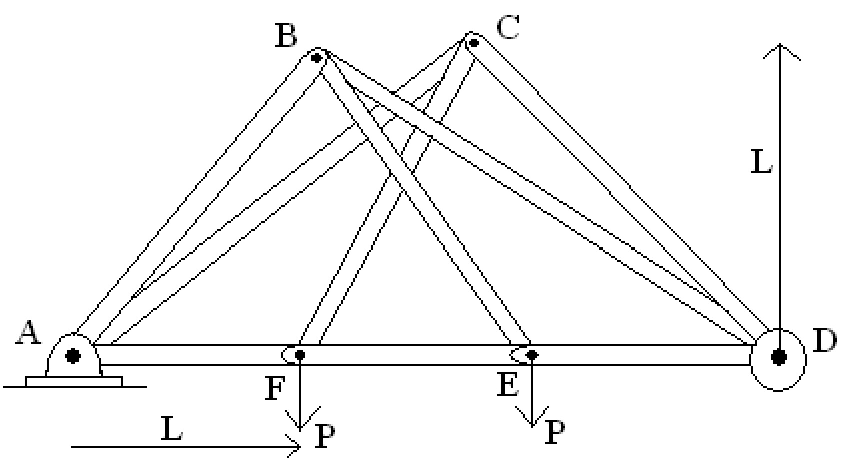
$P = \dfrac{2}{2.73} = 732.6N$

Compression$_{max}$ *= 1.58P*
1.2kN = 1.58P

$P = \dfrac{1.2}{1.58} = 759.4N$

P_{max} *= 732.6N ≈* ***732N*** *(at that point, member CA would be pulled apart)*

e.g. 3
Given: Dimensions of a double scissors truss and resultant forces at joint E and F.
Find: Force in each member.



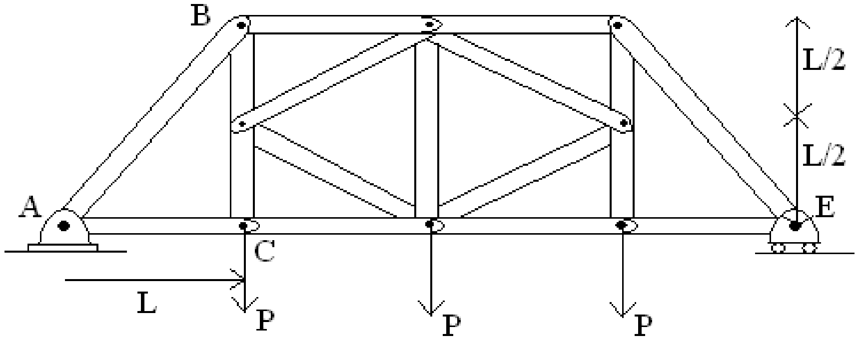
This problem is unusual because all joints have more than two unknowns, even if support reactions are known. However, the problem can be solved with the method of joints, by starting at joint E or F. $\sum F_{Ey} = 0$

$$\Rightarrow F_{EB} = \frac{P}{\sin 45°} \ \ (T)$$

Then, move on to joint B.

e.g. 4
Given: Dimensions of a K truss and resultant forces at H,G, and F shown below.
Find: Force in each member.

Here, we must calculate support reactions before we can do anything.



$R_A = R_Ay = R_E = \frac{3}{2}P$

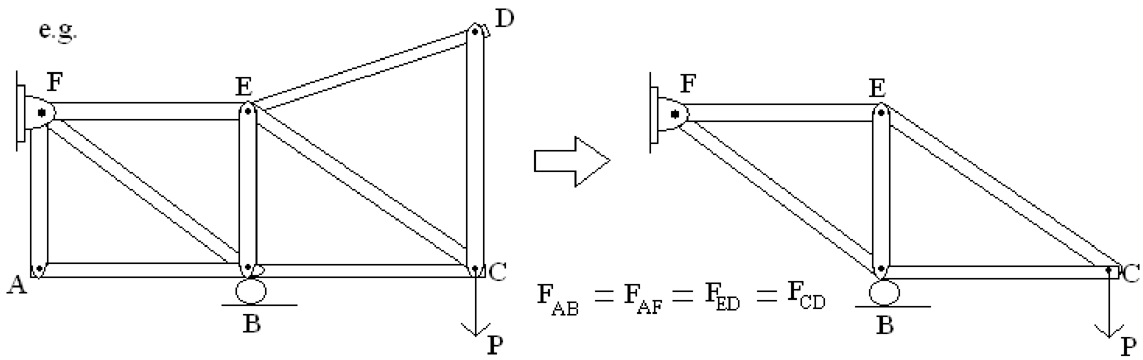
Start at joint A or E;
i.e., $\sum F_{Ay} = 0 \Rightarrow F_{AB} = \frac{3/2*P}{\sin 45°}$ *(C)*

$\sum F_{Ax} = 0 \Rightarrow F_{AC} = 3/2*P$ *(T)*

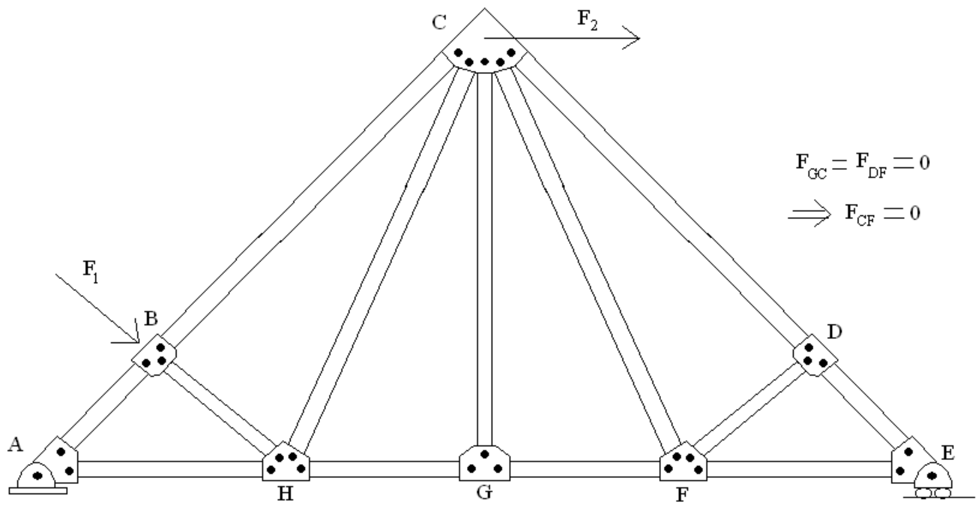
And continue from there to solve the problem.

<u>Zero Force Members</u>

If two elements meet at a joint and are *not* collinear and there is no external force at that joint, then they're both zero force members.



If three elements meet at a joint, two of which *are* collinear, and there is no external force at that joint, then the third member is a zero-force member.



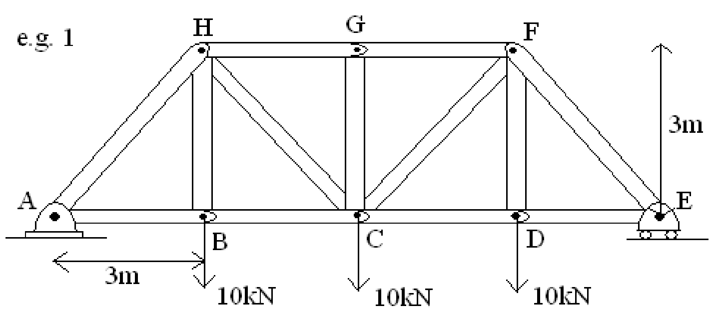
<u>The Method of Sections</u>

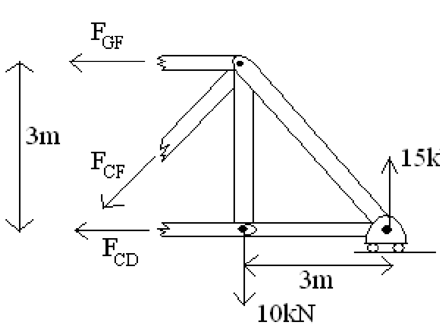
The method of sections is usually used to find the loadings on just a few beams, efficiently. The basic idea is that if a body is in equilibrium, then any part of the body is also in equilibrium. Rather than dismembering every beam and applying particle equilibrium at each joint, we can "slice" through the whole structure and apply rigid body equilibrium to either of the two portions. $\sum \mathrm{Fx} = 0, \sum \mathrm{Fy} = 0, \sum \mathrm{Mo} = 0$ must be satisfied for equilibrium. O can be anywhere in space. The other equations for equilibrium can also be used (see the beginning of the section "Equilibrium of a rigid body"). We should create a section with at least one known and at most three unknowns.

e.g. 1

Given: Bridge truss dimensions and equivalent forces at B, C, D.

Find: F_{GF}, F_{CF}, F_{CD}





Support reaction at E;

$$+ \quad \sum M_A : R_E(12) - 10(3) - 10(6) - 10(9) = 0$$

$$\Rightarrow R_E = 15kN$$

Member forces;

$$+ \quad \sum M_F : 15(3) - F_{CD}(3) = 0$$

$$+\uparrow \sum Fy : 15 - F_{CF}\cos 45° - 10 = 0$$

$$\xrightarrow{+} \sum Fx : -F_{GF} - 7.07\sin 45° - 15 = 0$$

F_{CD} = ***15kN (T)***

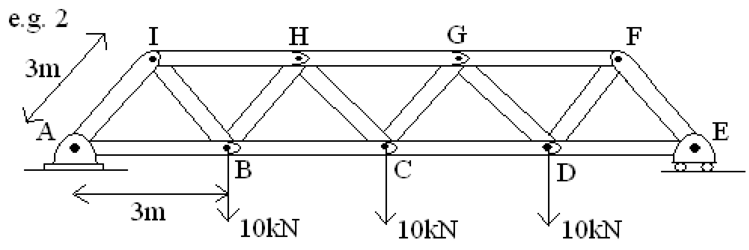
F_{CF} = ***7.07kN (T)***

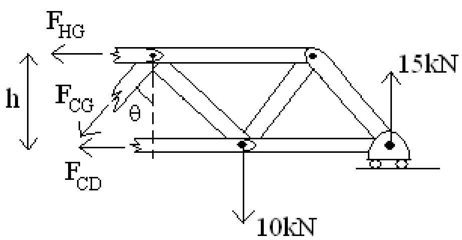
F_{GF} = ***20kN (C)***

e.g. 2

Given: Warren truss dimensions and equivalent forces at B, C, D shown below.

Find: F_{GH}, F_{CG}, F_{CD}





Support reaction at E;

R_E = 15kN (same as e.g. 1)

$\theta = \sin^{-1}(\frac{1.5}{3}) = 30°$

$h = \sqrt{3^2 - 1.5^2} = 2.60m$

$\sum M_G : 15(1.5+3) - 10(1.5)$
$- F_{CD}(2.60) = 0$

$+\uparrow \sum Fy : 15 - FCG\cos 30°$
$-10 = 0$

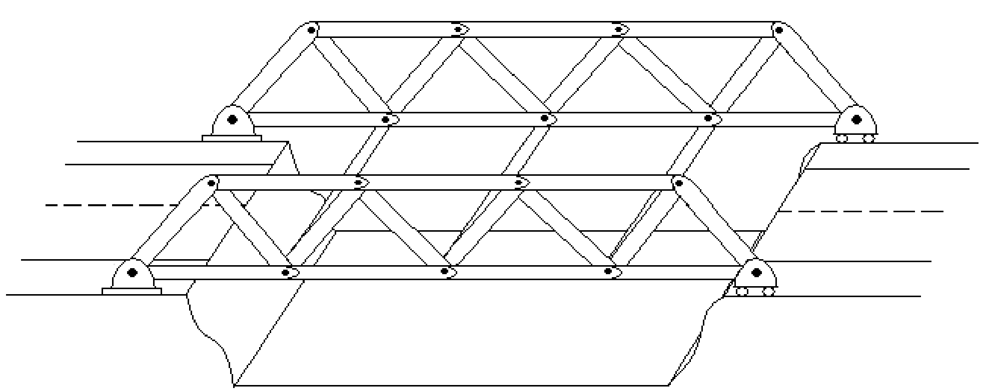
$\xrightarrow{+} \sum Fx : -F_{HG} - 5.77\sin 30°$
$-20.2 = 0$

*F_{CD} = **20.2kN (T)*** *F_{CG} = **5.77kN (T)*** *F_{HG} = -23.1 = **23.1kN (C)***

Comparing e.g. 1 and e.g. 2, the bridge seems to be more economical, for this kind of loading.

note: The method of joints and the method of sections can be easily applied to 3D "space trusses." For the method of joints, there would be 3 equations for particle equilibrium at each joint. For the method of sections, there would be 6 equations $\sum \text{Fx} = \sum \text{Fy} = \sum \text{Fz} = 0, \sum \text{Mx} = \sum \text{My} = \sum \text{Mz} = 0$.

Remember that one of our truss assumptions was that the loads were applied only at the joints. Our truss models pictured in the previous examples show loads only at the joints. But, in real-life, is this true? The answer is yes, as we can see in the diagram below.



The load from the road and vehicles rests entirely on the beams, as we can see in the exaggerated diagram. The beams are connected to the bottom joints of the trusses, so the load is transferred entirely to those bottom joints. Our assumption is thus valid. Finally, the truss rests on the ground at each end of the valley, so the load is entirely transferred to the ground. The beams connecting the top joints of the trusses are not shown but typically would be present.

It's always important not only to know how to analyze a *component* of a structural system, such as the truss examples we have done in this section, but also to understand the *load path* for the entire system. The above diagram and explanation described the load path for a truss bridge. The load path for a truss roof in a gymnasium would be different. We will take a more quantitative look at the load paths for a simple building in a different outline.

Hibbeler, R.C. Engineering Mechanics: Statics Tenth Edition. Pearson. Upper Saddle River, NJ 2004.
Johnson, Erik. Lecturer. Univ. of Southern California. CE205. Fall 2004.