## Moments

e.g.'s

(a)

(b)

(c)

(d)

Magnitude of moments about pivot $O$ :
(a) $F^{*} h$
(b) $F^{*} h \sin \theta$
(c) $F^{*}(L+h \cos \theta)$
(d) $F^{*} h$
note: the "cross product" can be used as well to determine the moment
The following examples essentially use the distributive law of cross products.
e.g. 1

e.g. 2

Given: Length and height of lever, magnitude of force.
Find: $\theta_{\text {max }}$, which yields the maximum moment about $O$, and $\theta_{\text {min, }}$, which yields the minimum moment.
e.g. 2

$M_{o}=(40 \cos \theta)(2)+(40 \sin \theta)(8)=$ $80 \cos \theta+320 \sin \theta=0=\left(M_{o}\right)_{\text {min }} \Rightarrow$ $\theta_{\text {min }}=2.90 \mathrm{rad} x \frac{180 \mathrm{deg}}{\pi \cdot \mathrm{rad}}=166^{\circ}$
note: $\tan ^{-1} \frac{2 f t}{8 f t}=14^{\circ}$ and $180^{\circ}-14^{\circ}=166^{\circ}$ which is what we found above.

In other words, if the line of action of the force passes through point $O$, then there is no moment ( $M_{o}=0$ ). This, of course, makes intuitive sense.
$M_{o}{ }^{\prime}=-80 \sin \theta+320 \cos \theta=0 \Rightarrow$
$\theta_{\max }=1.326 \mathrm{rad} x \frac{180 \mathrm{deg}}{\pi \cdot \mathrm{rad}}=76^{\circ} \Rightarrow$
$M o=330 \mathrm{lb}=\left(M_{o}\right)_{\max }$
note: the following example could be solved using the "triple scalar product". However, suffice to say, it is really the same as previous problems if the axes are rotated so that the z axis points into the page.
e.g.

Given: Torque of 80 lb *in required to loosen the nut $\left(M_{z}=80\right)$.
Dimensions and orientation of flex-head wrench given.
Find: Required force F to be applied to the end of the wrench.

$M_{z}=F d$
$d=.75+10 \sin 60^{\circ}$
$80=F^{*}\left(.75+10 \sin 60^{\circ}\right) \Rightarrow F=8.5 \mathbf{l b}$

Hibbeler, R.C. Engineering Mechanics: Statics Tenth Edition. Pearson. Upper Saddle River, NJ 2004.
Johnson, Erik. Lecturer. Univ. of Southern California. CE205. Fall 2004.

