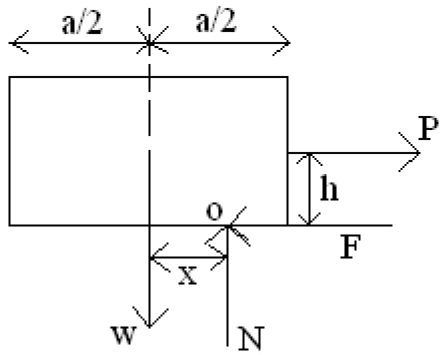
Friction

<u>Single contact block</u>



Moment equilibrium about point O is satisfied if Wx = Ph or $x = \frac{Ph}{W}$. The block will be on the verge of tipping if N acts at the right corner of the block.

Verge of tipping $\Rightarrow x = \frac{a}{2}$ check: $F \le \mu_s * N$

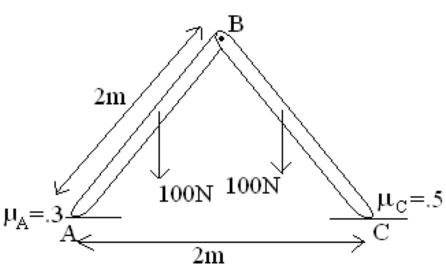
Equilibrium of the block also requires F = P

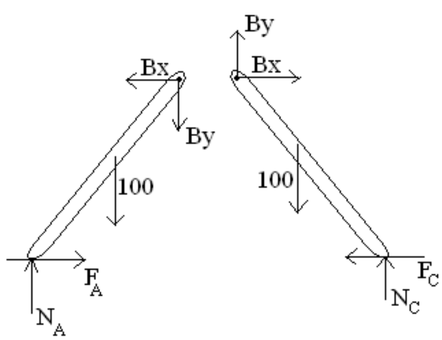
Verge of slipping $\Rightarrow F = \mu_s * N$ check: $x \le \frac{a}{2}$

<u>Multiple contact points</u>

Four possibilities:

1) No unknown external forces, *no* contact points necessarily have impending motion.

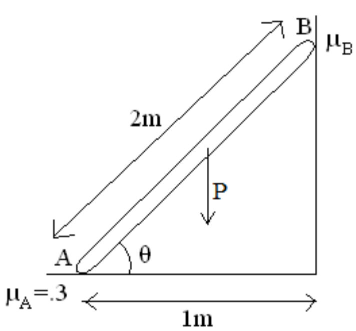


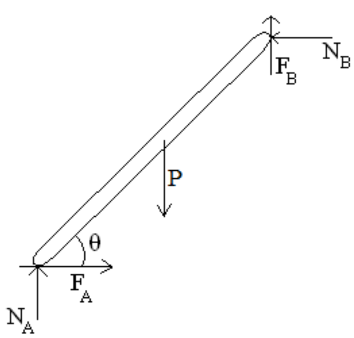


6 unknowns
6 equations (rigid body equilibrium)

check: $F_A \le \mu_A * N_A$
$F_C \le \mu_C * N_C$

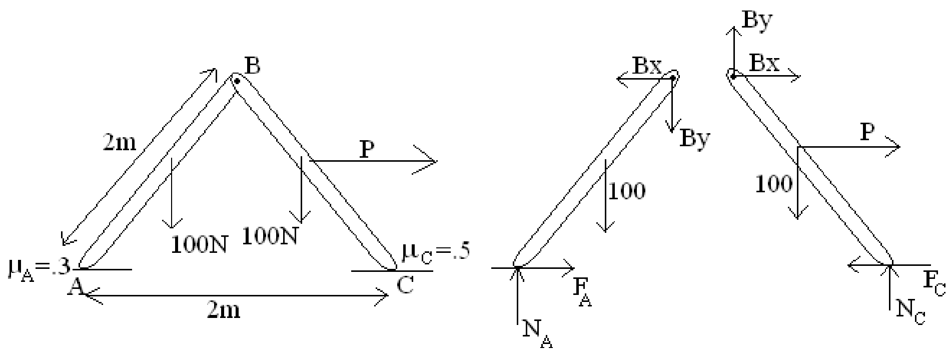
2) Maximizing an external force (minimizing an angle); *all* contact points on the verge of slipping.





5 unknowns
3 equations (rigid body equilibrium) + 2 equations ($F_A = \mu_A N_A$, $F_B = \mu_B N_B$)

3) Maximizing an external force, *some* (usually one) contacts points have impending motion (slipping or tipping)



7 unknowns
6 equations (rigid body equilibrium) + guess

- $F_A = \mu_A N_A$, if A slides first

check: $N_A \geq 0$

$F_C \leq \mu_C * N_C$

OR

- $F_C = \mu_C N_C$, if C slides first

check: $N_A \geq 0$

$F_A \leq \mu_A * N_A$

OR

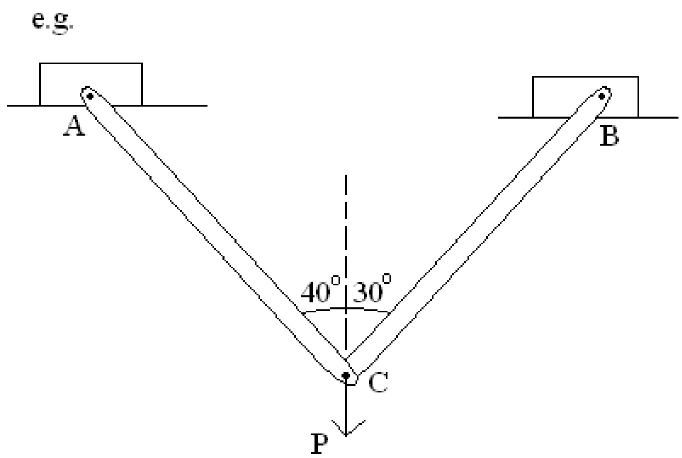
- $N_A = 0$, if rotation about C occurs first.

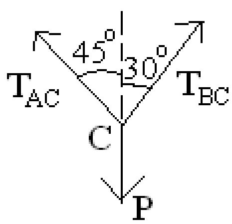
check: $F_A = 0$

$F_C \leq \mu_C * N_C$

4) Maximizing an external force; forces contribute solely to tipping.
(case 4 problems are fairly common – an example would be finding the maximum load a crane truck can hold before tipping)

e.g. (case 3)
Given: Vertical force P applied at connection. Mass of each block = 6 kg. μ_A *= .2* μ_B *= .8.*
Find: Largest force P before motion.





$$\xrightarrow{+} \sum Fx : T_{BC} \sin 30° - T_{AC} \sin 45° = 0$$

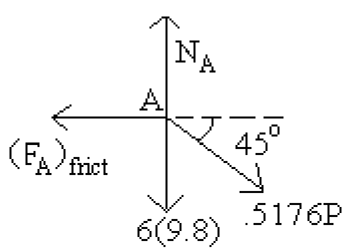
$$+\uparrow \sum Fy : T_{BC} \cos 30° + T_{AC} \cos 45° - P = 0$$

From $\sum Fx$, $T_{AC} = \dfrac{T_{BC} \sin 30°}{\sin 45°}$

Substitute into $\sum Fy$, $T_{BC} \cos 30° + \left(\dfrac{T_{BC} \sin 30°}{\sin 45°}\right) \cos 45° - P = 0$

$$T_{BC} = \frac{1}{\cos 30° + \sin 30° \cot 45°} P \approx .732P$$

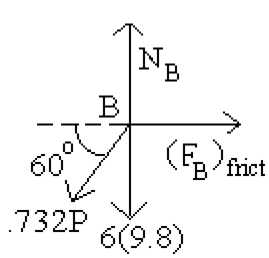
$$T_{AC} = \frac{\sin 30°}{\sin 45°(\cos 30° + \sin 30° \cot 45°)} P \approx .5176P$$



$$+\uparrow \sum Fy : N_A - 6(9.8) + .5176P \sin 45° = 0$$

$$\Rightarrow N_A = 58.86 + .366P$$

$$\xrightarrow{+} \sum Fx : .5176P \cos 45° - (F_A)\, frict = 0 \Rightarrow (F_A)\, frict = .366P$$



$$+\uparrow \sum Fy : N_B - 6(9.8) + .732P \sin 60° = 0 \Rightarrow N_B = 58.86 + .634P$$

$$\xrightarrow{+} \sum Fx : .(F_B)\, frict - .732P \cos 60° = 0 \Rightarrow (F_B)\, frict = .366P$$

Guess: $(F_A)_{frict} = \mu_A N_A$
$.366P = .2(58.86+.366P) \Rightarrow P_{max} = \mathbf{40.2N}$

Check: $(F_B)_{frict} = .366(40.2) = 14.7N$
$\mu_B N_B = .8[58.86+.634(40.2)] = 67.5N$
$(F_B)_{frict} = 14.7 \leq \mu_B N_B = 67.5$ *OK*

note: could have broken apart the frame, applied rigid body equilibrium, and made our guess at the beginning or the end, in that manner. This would have been more difficult though since member lengths were not given.

Hibbeler, R.C. Engineering Mechanics: Statics Tenth Edition. Pearson. Upper Saddle River, NJ 2004.
Johnson, Erik. Lecturer. Univ. of Southern California. CE205. Fall 2004.