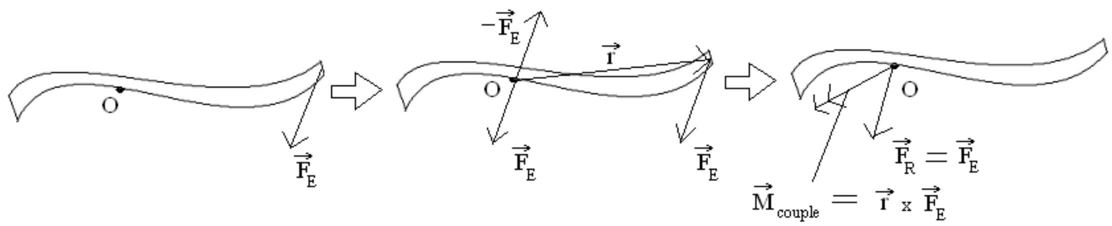
Equivalent force for rigid body



By looking at the above diagram from right to left, we can see that a force and a moment can be reduced to a single force located at a certain distance.

If there are multiple forces, then $F_{Rx} = \sum Fx$ and $F_{Ry} = \sum Fy$.

$\vec{F}_R = < F_{Rx}, F_{Ry} >$

$\vec{M}_{Ro} = \sum Mo$

note: the magnitude and direction of $\vec{F}_R$ is independent of the location of O. But, $\vec{M}_{Ro}$ depends upon the position vectors $\vec{r}$ and therefore depends upon the location of O.

$\vec{F}_E$ would have the same magnitude as $\vec{F}_R$ but would be located at a distance d= $\vec{M}_{Ro} \div \vec{F}_{R,}$ a distance from the chosen point O.

note: $\vec{F}_E$ and its line of action is the only possible $\vec{F}_E$ for a given system. *Where* you take the moment determines the magnitude and location of d, but it always puts $\vec{F}_E$ on the same line of action.

e.g. 1
Given: F_1 = 2 lb, F_2 = 1 lb, F_3 = 3 lb
Find: equivalent force F_E and location d of this force
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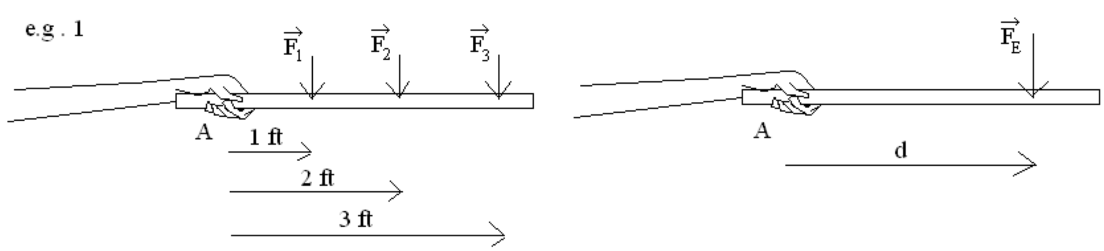
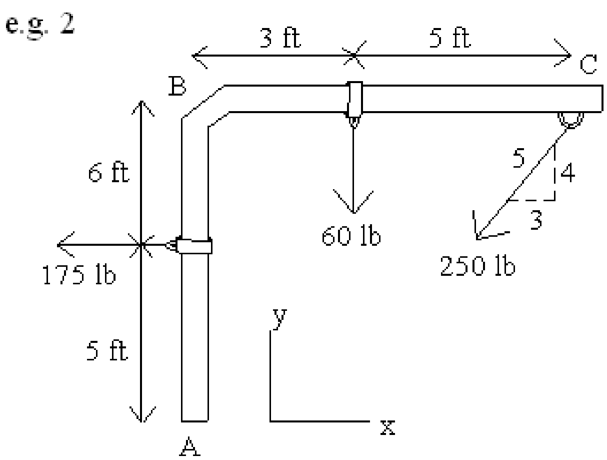


*M_A = 2*1+1*2+3*3 = 13 lb*ft*
(this moment would be resisted by the person's fingers and palm of hand)

$F_E = F_R = 2+1+3 =$ ***6 lb***

$F_E * d = M_A \Rightarrow d = \frac{13}{6} =$ ***2.1667 ft***

e.g. 2
Given: Three forces acting at the locations shown.
Find: Distance from A to the point where the line of action of $\vec{F}_E$ intersects $\overline{AB}$.
Distance from B to the point where the line of action of $\vec{F}_E$ intersects $\overline{BC}$.

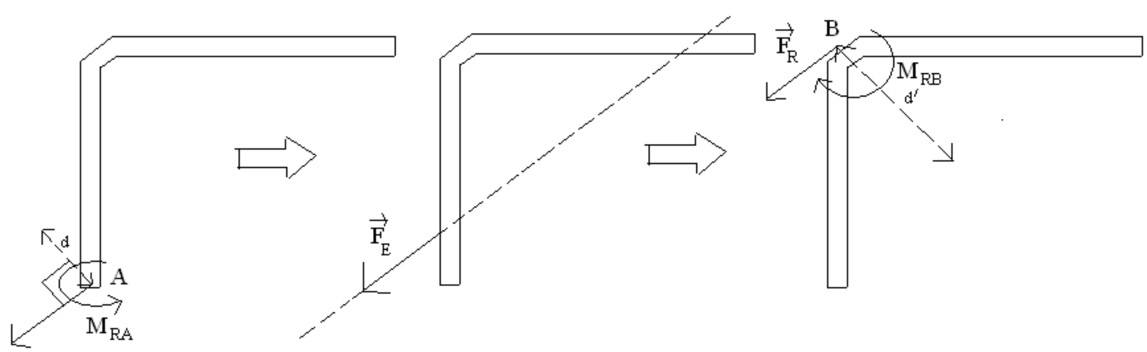


$$\sum Fx = -175 - 250 * \frac{3}{5} = -325lb$$

$$\sum Fy = -60 - 250 * \frac{4}{5} = -260lb$$

$$|F_R| = \sqrt{325^2 + 260^2} = 416lb$$

note:



$$\sum M_{RA} = 175*5+250*\frac{3}{5}*(6+5) - 60(3) - 250*\frac{4}{5}*(5+3) = 745lb*ft$$

$$d = \frac{745}{416} = 1.79\,ft$$

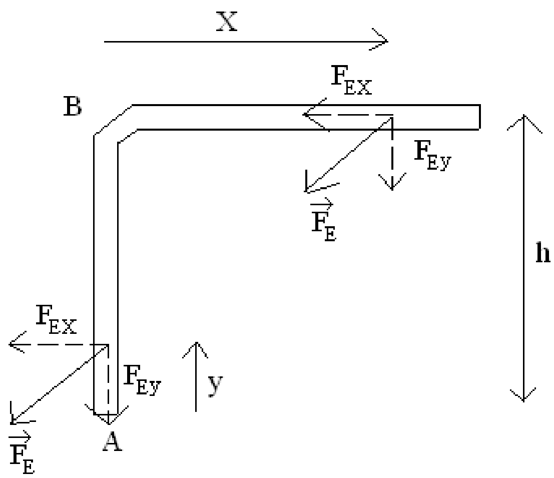
OR

$$\circlearrowleft + \quad \sum M_{RB} = -175*6-60(3)-250*\frac{4}{5}*(3+5) = -2830 lb*ft$$

$$d' = \frac{-2830}{416} = -6.8\,ft$$

Both solutions seem to place $\vec{F}_E$ *along our expected line of action, as the above illustration shows.*

Going back to the original question and solution $\rightarrow$



Looking first at the lower left portion:

$F_{EX}*y + F_{EY}*0 = 745$ *(found from* $\sum M_{RA}$*)*

$\Rightarrow y =$ ***2.29 ft***

Now looking at the upper right portion:

$F_{EX}*h - F_{EY}*x = 2830 \Rightarrow x =$ ***10.9 ft***

Hibbeler, R.C. Engineering Mechanics: Statics Tenth Edition. Pearson. Upper Saddle River, NJ 2004.
Johnson, Erik. Lecturer. Univ. of Southern California. CE205. Fall 2004.