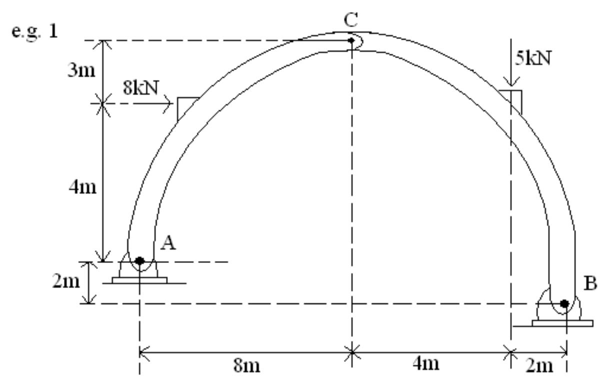
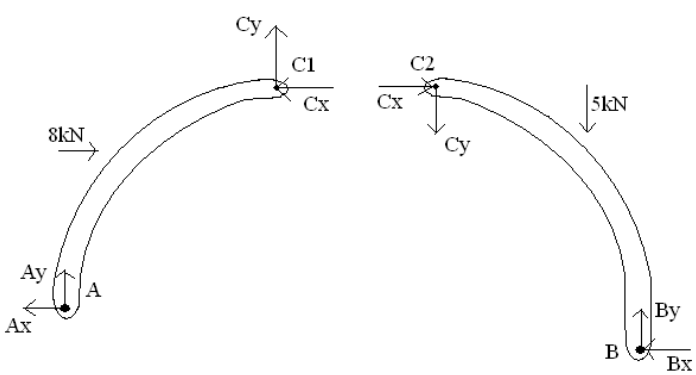
Equilibrium of a rigid body + Newton's 3rd Law

We've already analyzed frames and machines in the section "Equilibrium of a rigid body", but if we apply the same principle that was used for the "method of sections" – if a body is in equilibrium then any part of the body is also in equilibrium – then frames and machines (or any kind of structure) can be broken into pieces and analyzed using $\sum Fx = 0, \sum Fy = 0, \sum Mo = 0$ (or the other equations for rigid body equilibrium) for each piece.

Important: Forces common to any two contacting pieces must act with equal and magnitudes and opposite sense on the respective members (Newton's 3rd Law). These two principles – equilibrium for each part, and equal and opposite forces between parts – enables us to find support reactions when there are fewer "knowns" than in previous problems in this chapter, and now we can find internal reactions which we could not do at all before.

e.g. 1
Given: Three-hinged arc with external forces applied at locations shown.
Find: Reactions at A, B, C.





3 equations per free-body diagram
$\Rightarrow$ *6 total equations = 6 unknowns*

note: If there was an external force at C, it would go on either the right piece or the left, but NOT both

left side:

$$\overset{+}{\circlearrowleft} \sum M_{C1} : -Ay(8) - Ax(4+3) + 8(3) = 0$$

$$\overset{+}{\circlearrowleft} \sum M_{A} : -8(4) + Cx(4+3) + Cy(8) = 0$$

$$+\uparrow \sum Fy : Ay + Cy = 0$$

right side: $\circlearrowleft + \sum M_{C2} : By(4+2) - Bx(2+4+3) - 5(4) = 0$

$\circlearrowleft + \sum M_B : Cy(4+2) - Cx(2+4+3) + 5(2) = 0$

$+\uparrow \sum Fx : Cx - Bx = 0$

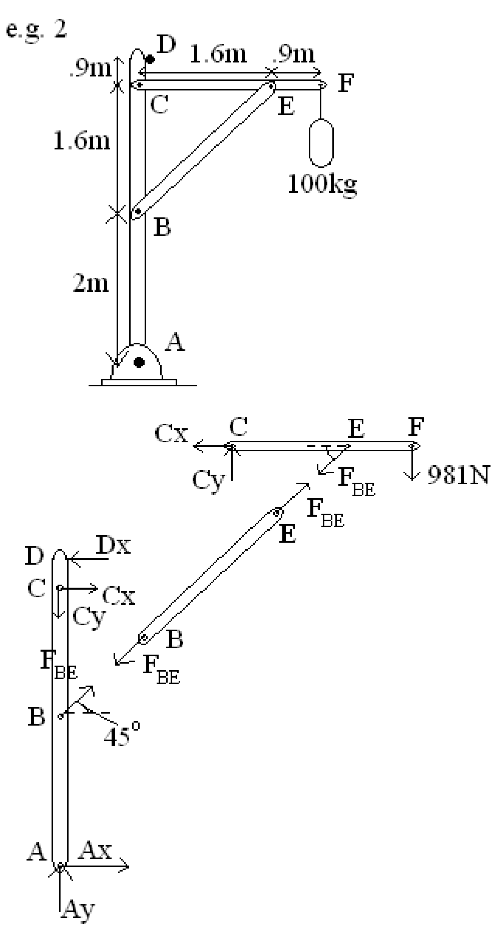
From $\sum M_B$, $Cy = \dfrac{Cx(2+4+3) - 5(2)}{4+2}$

Substitute into $\sum M_A$, *Cx =* ***2.386kN*** *Cy =* ***1.91kN*** *Bx =* ***2.386kN*** *By =* ***6.91kN***

Ay = -1.91 $\Rightarrow$ *Ay =* ***1.91kN (down)*** *Ax =* ***5.6kN***

note: As the structures become more complex, it is easy to see that a calculator which can solve systems of linear and nonlinear equations simultaneously, can come in quite handy. The TI-89 calculator is one such calculator which has this capability.

e.g. 2
Given: Frame supported by a hinge at A and a roller at D. 100kg mass at F.
Find: Support reactions and all internal reactions at labeled joints. (see pic below)



entire frame:

$\circlearrowleft + \sum M_A : -981(2.5) + Dx(4.5) = 0 \Rightarrow Dx =$ ***545N***

$\xrightarrow{+} \sum Fx : Ax - Dx = 0 \Rightarrow Ax =$ ***545N***

$+\uparrow \sum Fy : Ay - 981 = 0 \Rightarrow Ay =$ ***981N***

horizontal member:

$\circlearrowleft + \sum M_C : -981(2.5) - F_{BE} \sin 45°(1.6) = 0$

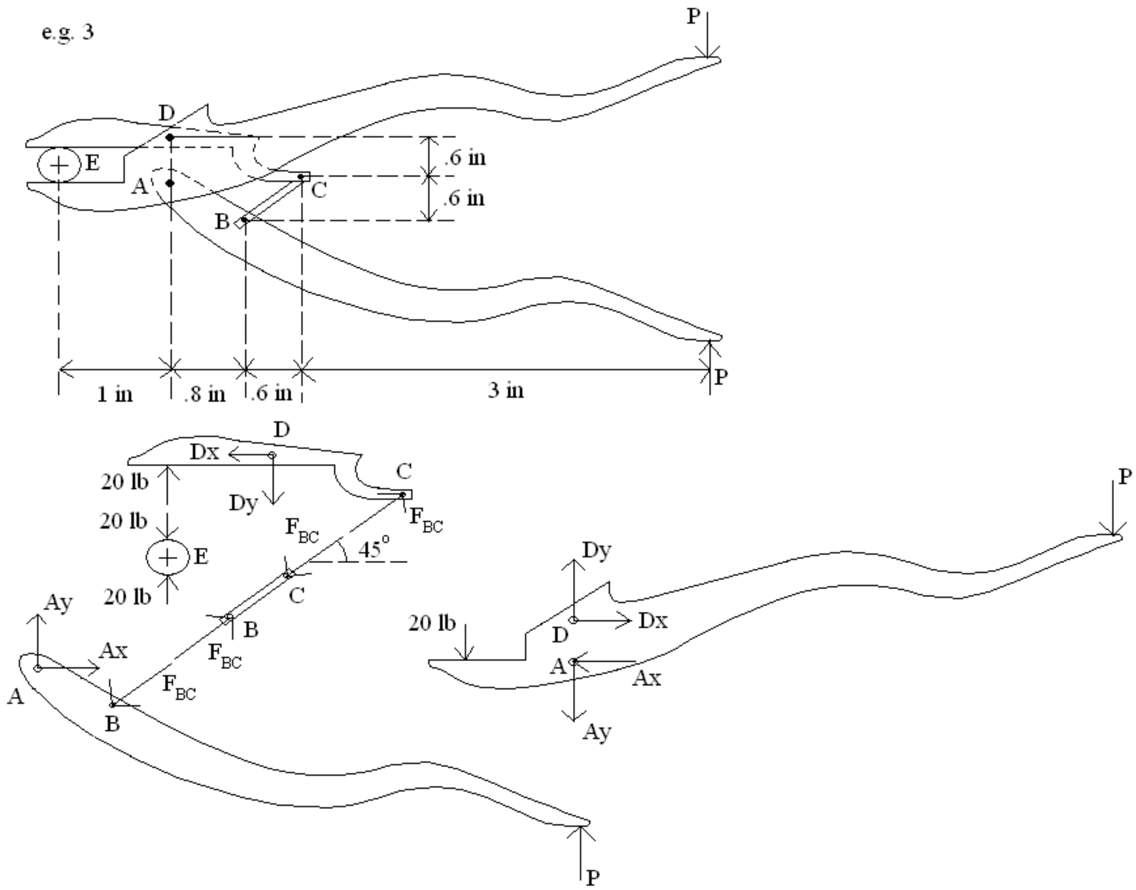
$\xrightarrow{+} \sum Fx : -Cx - F_{BE} \cos 45° = 0$

$+\uparrow \sum Fy : Cy - 981 - F_{BE} \sin 45° = 0$

F_{BE} *= -2168* $\Rightarrow$ F_{BE} *=* ***2168N (C)***

Cx = ***1533N*** *Cy = -552* $\Rightarrow$ *Cy =* ***552N (down)***

e.g. 3
Given: Dimensions of pruning shears. We need a cutting (normal/contact) force of 20 lb.
Find: Squeezing force P applied at location shown below. (see pic below)



upper blade:

$$\circlearrowleft + \sum M_D : F_{BC} \sin 45°(1.4) + F_{BC} \cos 45°(.6) - 20(1) = 0 \Rightarrow F_{BC} = \mathit{14.14\ lb}$$

$$\xrightarrow{+} \sum Fx : F_{BC} \cos 45° - Dx = 0 \Rightarrow Dx = \mathit{10.0\ lb}$$

lower jaw/upper arm:

$$\circlearrowleft + \sum M_A : 20(1) - 10(.6) - P(3 + .6 + .8) = 0 \Rightarrow P = \boldsymbol{3.2\ lb}$$

note: started with a piece that had at least one known and at most three unknowns.
note: these pruning shears enable one to multiply their grip force by over 5 times.

Hibbeler, R.C. Engineering Mechanics: Statics Tenth Edition. Pearson. Upper Saddle River, NJ 2004.

Johnson, Erik. Lecturer. Univ. of Southern California. CE205. Fall 2004.