## Equilibrium of a rigid body

So far, we've looked at types of problems which are mostly academic. Practical engineering problems involve taking applied forces and using engineering methods to calculate how those applied forces are distributed among the elements of your structure. This was discussed in the first chapter, titled "Introduction to Structural Engineering." Now that we have the needed foundation, we will begin to look at more practical-type engineering problems.

Previously, we've seen that $\vec{F}_{\text {E }}$ creates a different moment depending on the location of O . In the previous example, shown to the right, we summed moments about point A to find the location of $\vec{F}_{\text {E. }}$ This moment is resisted by the connection at A, otherwise the whole structure would clearly tip over. In the example pictured to the right, there are actually three "support
 reactions" at point A. There are two forces, along with the resisting moment described two sentences ago. These three support reactions combine to create a vector that is exactly equal and opposite $\vec{F}_{\text {E. Thus, }}$ the structure as a whole is static. So, we have three unknowns at point A, and we need three equations.
$\sum \mathrm{Fx}=0 \quad$ O may lie either on or off the body and can be a point or an axis.
$\sum \mathrm{Fy}=0 \quad \mathrm{OR}, \quad \sum \mathrm{M}_{\mathrm{A}}=0 \mathrm{~A}, \mathrm{~B}$, and C must not lie on the same line.
$\sum \mathrm{Mo}=0 \quad \sum \mathrm{M}_{\mathrm{B}}=0$

Alternatively,

$$
\begin{aligned}
& \sum \mathrm{F}_{\mathrm{a}}=0 \\
& \sum \mathrm{M}_{\mathrm{A}}=0 \\
& \sum \mathrm{M}_{\mathrm{B}}=0
\end{aligned}
$$ A line passing through points A and B must NOT be perpendicular to the "a" axis. If they were allowed to be perpendicular, then there could be $\vec{F}_{\mathrm{R}} \neq 0$ which is perpendicular to the "a" axis ( $\sum \mathrm{F}_{\mathrm{a}}=0$ ) and has a line of action along $\overline{\mathrm{AB}}\left(\sum \mathrm{M}_{\mathrm{A}}=\sum \mathrm{M}_{\mathrm{B}}=0\right)$. To prevent this, the condition can simply be stated that $\overline{\mathrm{AB}}$ cannot be perpendicular to the "a" axis.

e.g. 1

Find: Support reactions (see pic below)

$\sum F x: 600 \cos 45-B x=0 \Rightarrow B x=424 N$
$\sum M_{B}: 100(2)+(600 \sin 45)(5)-A y(7)-(600 \cos 45)(.2)=0 \Rightarrow A y=319 N$
$\sum F y: A y-600 \sin 45-100-200+B y=0 \Rightarrow B y=405 N$
OR
$\sum M_{A}: B y(7)-(600 \sin 45)(2)-(600 \cos 45)(.2)-100(5)-200(7)=0 \Rightarrow B y=405 N$
$\sum F x=0$
note: $\overline{A B}$ not perpendicular to the x axis $\Rightarrow \sum M_{B}=0$ can be used.

$$
\sum M_{A}=0
$$

e.g. 2

Given: The weight of each book is $w$ and each has length " $a$ ".
Find: Maximum distance d that the top book can extend over the bottom book, before the stack topples. (see pic below)


Half of the book's length can hang out over B. Need to find the distance that the second book can hang over $A$.

$$
d_{2}+x+d_{1}=\frac{a}{2}+x+d_{1}=a \Rightarrow x=a-\frac{a}{2}-d_{1}=\frac{a}{2}-d_{1}
$$

[normal(contact) forces not shown]

tip over $\Rightarrow M_{\text {top }}=M_{\text {mid }}$

$$
w_{\text {middle }}(x)-w_{\text {top }}\left(d_{1}\right)=0
$$

$$
w\left(\frac{a}{2}-d_{1}\right)-w\left(d_{1}\right)=0
$$

$$
\frac{a}{2}=2 d_{1} \Rightarrow d_{1}=\frac{a}{4} \Rightarrow d=d_{1}+d_{2}=\frac{a}{4}+\frac{a}{2}=\frac{3 a}{4}
$$

e.g. 3

Given: (see pic) Spring constant $k=40 \mathrm{~N} / \mathrm{m}$. Spring is compressed $.2 m$. Angle of slope $30^{\circ}$ as shown.
Find: Support reactions at $A$ in terms of $F_{A x}$ and $F_{A y}$, and magnitude of resultant force at wheel bearing $B$.

$\longrightarrow \sum F x: F_{A x}-F_{B} \cos 60^{\circ}=0$
$+\uparrow \sum F y: F_{A y}+F_{B} \sin 60^{\circ}-8=0$
$\varliminf_{+}^{+} \sum M_{B}: 8(.125)-F_{A y}(.275)-F_{A x}(.1)=0$
From $\sum F x, F_{A x}=F_{B} \cos 60^{\circ}$
From $\sum F y, F_{A y}=8-F_{B} \sin 60^{\circ}$
Substitute into $\sum M_{B} \Rightarrow$
$F_{B}=6.38 N \quad F_{A x}=3.19 N \quad F_{A y}=2.47 N$

## Two and Three Force Members

When forces are applied at only two points on a member, $\vec{F}_{\mathrm{A}}$ must be of equal magnitude ad opposite direction to $\vec{F}_{\text {в }}$ (see pic). The line of action is along $\overline{\mathrm{AB}}$.


If a member is subjected to forces at only three points, then $\vec{F}_{1}, \vec{F}_{2}$, and $\vec{F}_{3}$ must be either concurrent:

or parallel:


Knowing about two and three force members can sometimes simplify calculations and can also serve as a visual check for free-body diagrams which you think contain forces at only a few locations.
e.g.

Given: An external force resisted by a two-force link and a pin. (see pic)
Find: The magnitude of the support reactions $F_{A}$ and $F_{B}$ and their directions $\theta_{A}$ and $\theta_{B}$. (see below)

$\theta_{B}=\tan ^{-1} \frac{.2}{.2}=45^{\circ}$
$\theta_{A}=\tan ^{-1} \frac{.7}{.4}=60.3^{\circ}$
Looking at the free body diagram of the three force member:
$\sum F x: F_{A} \cos 60.3^{\circ}+400-F_{B} \cos 45^{\circ}=0$
$\sum F y: F_{A} \sin 60.3^{\circ}-F_{B} \cos 45^{\circ}=0$
Solving $\rightarrow F_{A}=1.07 \mathrm{kN} \quad F_{B}=1.32 \mathrm{kN}$

Hibbeler, R.C. Engineering Mechanics: Statics Tenth Edition. Pearson. Upper Saddle River, NJ 2004.
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