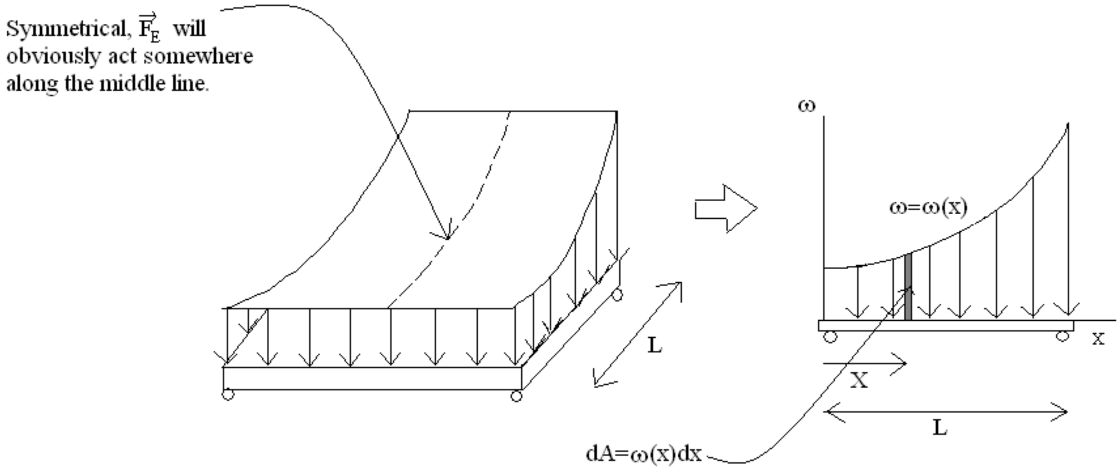
Distributed loading

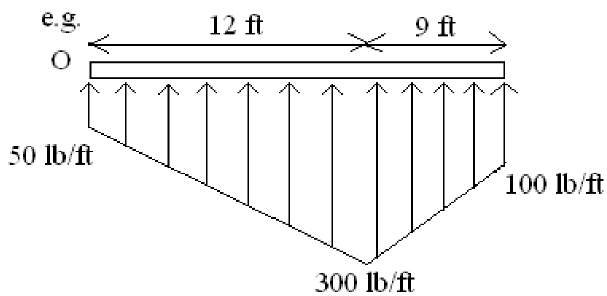


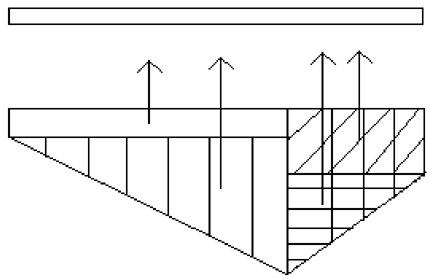
$F_R = A = \int_L \omega(x)dx$ $\qquad M_{RO} = \int_L x\omega(x)dx$

$$\text{“d”} = \bar{x} = \frac{\int_L x\omega(x)dx}{\int_L \omega(x)dx}$$

$\omega(x) =$ density function = force per unit length (i.e. $40(x^2+2)$ could be $\omega(x)$ for the ramp above). For uniform density, the magnitude of the constant (i.e. $\omega(x) = 40$) affects F_R but not $\bar{x}$.

e.g.
Given: Density at the three vertices of the triangular + rectangular distribution.
Find: F_R and its location measured from O, for equivalency.
(see pic below)





$$\sum F = lw + \frac{1}{2}BH + lw + \frac{1}{2}BH = 12(50) + \frac{1}{2}(300-50)(12) + 9(100) + \frac{1}{2}(300-100)(9)$$

$$= \mathit{3900lb} = \boldsymbol{F_R}$$

$$\sum M = 12(50)(6) + \frac{1}{2}(300-50)(12)(8) + 9(100)(12+4.5) + \frac{1}{2}(300-100)(9)(12+3) = 43950 lb * ft$$

$$= \boldsymbol{M_o}$$

$$\bar{x} = \frac{43950}{3900} = \boldsymbol{11.27\ ft}$$

note: For a triangular distribution, the equivalent force $\frac{1}{2}\mathrm{BH}$ is located a distance along its base (from the peak side) equal to $\frac{1}{3}$ length .