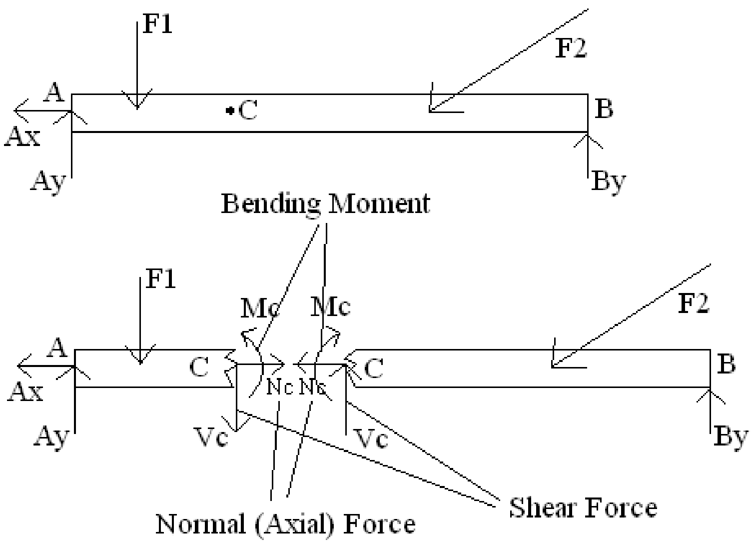
Beams – bending moment diagrams and shear force diagrams

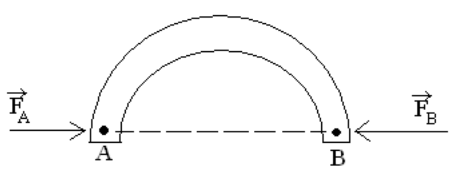
External forces on a frame create “internal” forces at each connection, and they also create internal forces within each member, which tend to deform it.



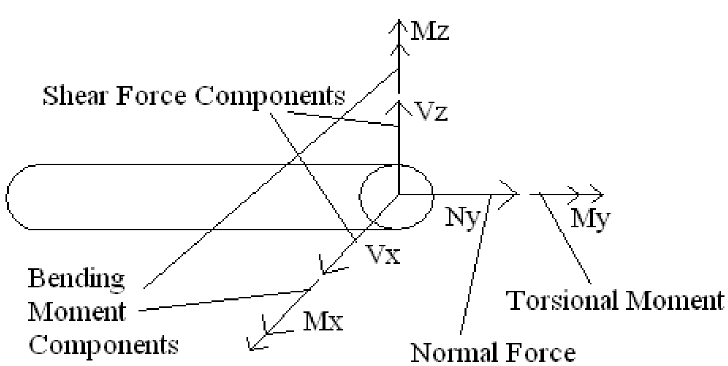
Nc is an axial (tensile or compressive) force, which is a type of force we’ve already dealt with. Note equal and opposite forces on each piece of the beam, from Newton’s 3rd Law.

Shear and bending moments within a beam are non-zero if:

1) The beam is a two force member but is not straight.



2) The beam is subjected to multiple forces (other than at the joints).



note: The sense of V and M drawn to the left and above is treated as positive in structural engineering.

Cutting Method

We will now develop methods for drawing shear force and bending moment diagrams that illustrate shear and moment magnitudes at every point along a beam. We will not be concerned with axial forces, since all external forces will act perpendicular to the members.
Picking the right size beam requires knowledge of V and M at each point along the beam's axis. As we will see in the next outline "Mechanics of Materials" and the following outlines on concrete and steel design, one can then use appropriate formulas to determine the required cross sectional area. Graphs of V and M as functions of x are called shear force diagrams (SFD) and bending moment diagrams (BMD).
The cutting method is useful for: *simple* distributed loads where a complicated w(x) function is not given.

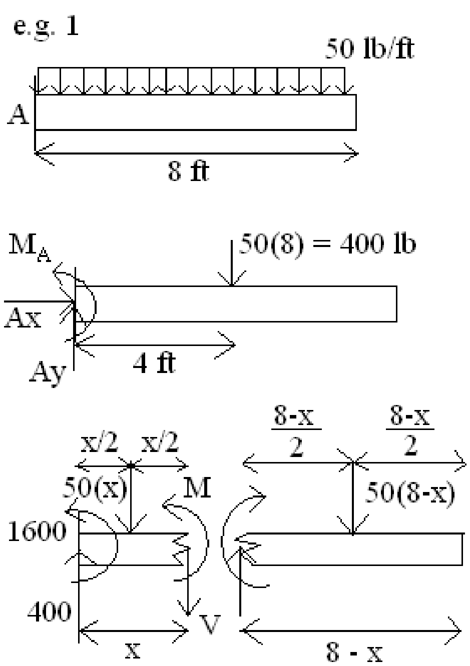
Process:

1) Find support reactions
2) Find equations for V and M from equilibrium equations
3) Find peak values

note: signs are important!
note: assumed sense of V and M is important!

Look over the following examples to fully understand this method.

e.g. 1
Given: 8 ft beam connected with fixed support and subjected to distributed load shown.
Find: SFD and BMD for the beam.



Support reactions:

Ax = 0

$$+\uparrow \sum Fy : Ay - 400 = 0 \Rightarrow Ay = 400lb$$

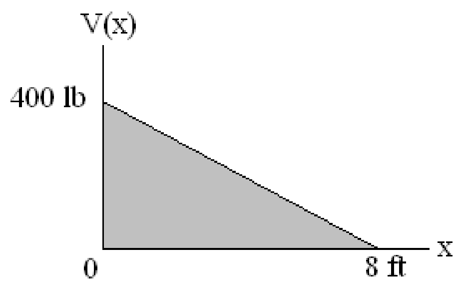
$$+ \circlearrowleft \sum M_A : M_A - 400(4) = 0 \Rightarrow M_A = 1600lb * ft$$

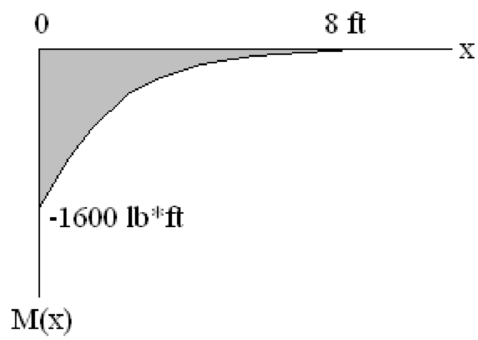
magnitude and location of distributed load on each piece must be in terms of x as shown.

$$+\uparrow \sum Fy : 400 - 50(x) - V = 0 \Rightarrow V = 400 - 50(x)$$

$$+ \circlearrowleft \sum M_{cut} : 1600 - 400(x) + 50(x)(\frac{x}{2}) + M = 0 \Rightarrow$$

$$M = 400x - 25x^2 - 1600$$



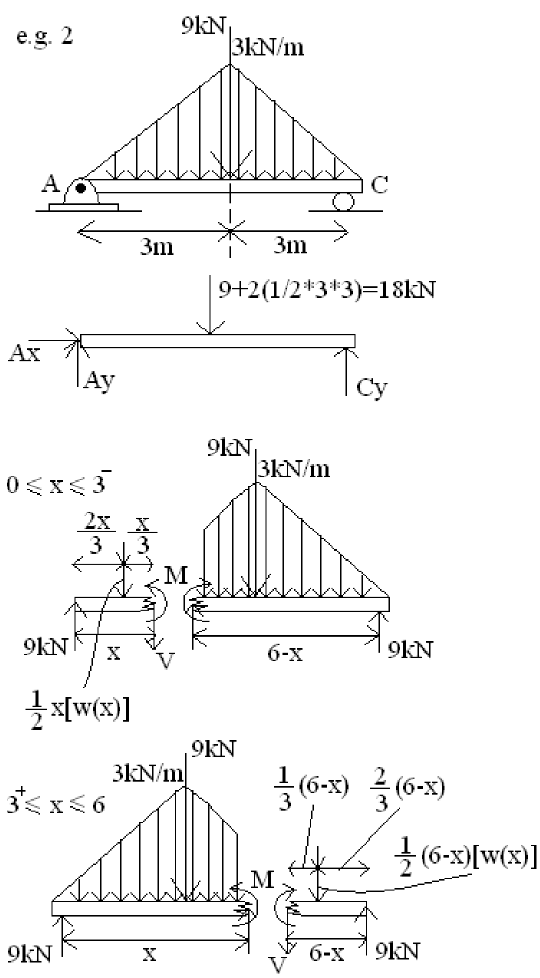


note: Fixed beams like this are sometimes tapered (thicker near the support) in order to be cost effective, since we can see that the forces are large only near the support.

e.g. 2

Given: Beam supported at both ends. Distributed load and concentrated force (see pic below).

Find: SFD and BMD for the beam.



Support reactions:
Ax = 0
Ay = Cy = 9kN (symmetry)

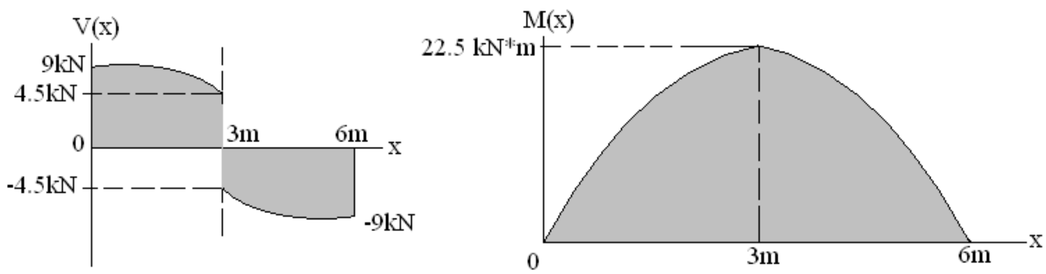
$0 \leq x \leq 3^-$

choose left side: from similar triangles (or y = mx + b), $\frac{w(x)}{x} = \frac{3kN/m}{3m} \Rightarrow w(x) = 1\frac{kN}{m^2}(x)$

$$\uparrow \sum Fy: 9 - \frac{1}{2}x^2 - V = 0 \Rightarrow V = 9 - \frac{1}{2}x^2$$

$$\overset{+}{\circlearrowleft} \sum M_{cut}: 9(6-x) - \frac{1}{2}(6-x)^2[\frac{1}{3}(6-x)] - M = 0$$

$$\Rightarrow M = 9(6-x) - \frac{1}{6}(6-x)^3$$



Graphical Method

This method is useful for: Anytime, but especially for complicated distributed loads where w(x) is give as a function

Process:

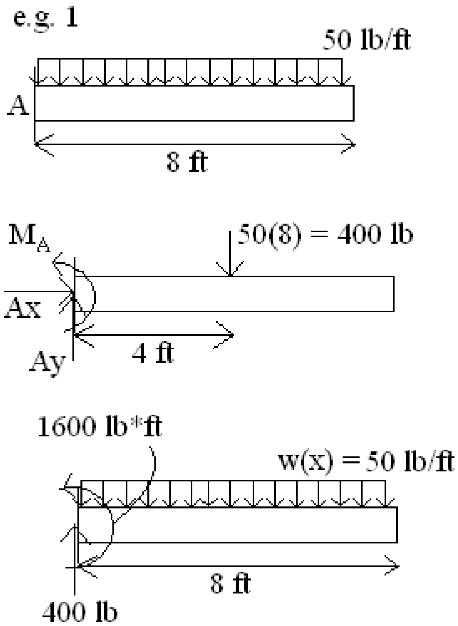
1) Find support reactions
2) Find equations for V and M using integration
3) Find peak values

$V(x) = -\int w(x)dx$ $\qquad$ $M(x) = \int V(x)dx$

note: A point load changes V(x) AT THAT POINT by the amount of the point load. An external moment changes M(x) AT THAT POINT by the amount of the external moment. Work from left to right! Assumed sense of V and M is still important, and w(x) is positive in downward direction.

Look over the following examples to fully understand this method.

e.g. 1
Given: 8 ft beam connected with fixed support and subjected to distributed load shown.
Find: SFD and BMD for the beam.



Support reactions:

Ax = 0

$$+\uparrow \sum Fy : Ay - 400 = 0 \Rightarrow Ay = 400lb$$

$$+ \sum M_A : M_A - 400(4) = 0 \Rightarrow M_A = 1600lb * ft$$

$$x = 0^- : V(0^-) = 0 \quad M(0^-) = 0$$

$$x = 0^+ : V(0^+) = V(0^-) + 400 = 400$$

$$M(0^+) = M(0^-) - 1600 = -1600$$

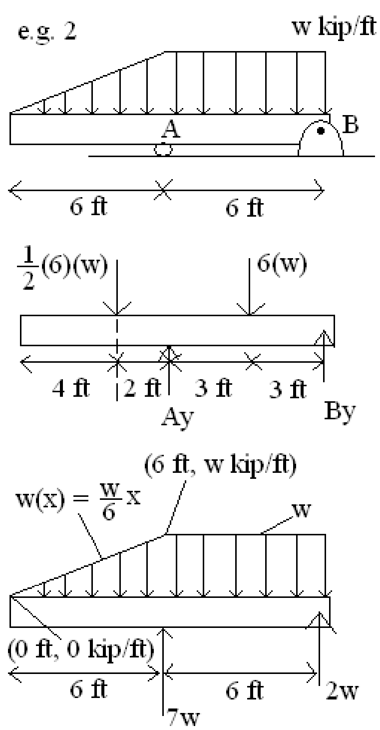
$$0^+ \le x \le 8 : V(x) = V(0^+) - \int_0^x 50dx = 400 - 50x$$

$$M(x) = M(0^+) + \int_0^x 400 - 50xdx$$

$$= -1600 + 400x - 25x^2$$

(compare with e.g.1 for "cutting method")

e.g. 2
*Given: Beam shown will fail for M>30 kip*ft or V>8kip at any point.*
Find: Largest distributed load w possible (see pic below).



Support reactions:

$$+ \sum M_B : 6(w)(3) + \frac{1}{2}(6)(w)(8) - Ay(6) = 0 \Rightarrow Ay = 7w$$

$$\uparrow \sum Fy : 7w - \frac{1}{2}(6)(w) - 6(w) + By = 0 \Rightarrow By = 2w$$

$$x = 0^- : V(0^-) = 0 \quad M(0^-) = 0$$

$$x = 0^+ : V(0^+) = V(0^-) + 0 = 0 \quad M(0^+) = 0$$

$$0^+ \le x \le 6^- : V(x) = V(0^+) - \int_0^x \frac{w}{6} xdx = -\frac{wx^2}{12}$$

$$M(x) = M(0^+) + \int_0^x -\frac{wx^2}{12} dx = -\frac{wx^3}{36}$$

$$x = 6^- : V(6^-) = -\frac{w(6)^2}{12} = -3w \quad M(6^-) = -\frac{w(6)^3}{36} = -6w$$

$$x = 6^+ : V(6^+) = V(6^-) + Ay = -3w + 7w = 4w$$

$$M(6^+) = M(6^-) + 0 = -6w$$

$$6^+ \le x \le 12^- : V(x) = V(6^+) - \int_6^x w dx = 4w - w(x-6) = 10x - wx$$

$$M(x) = M(6^+) + \int_6^x 10w - wx dx = -6w + 10wx - \frac{wx^2}{2} - [10w(6) - \frac{w(6)^2}{2}]$$

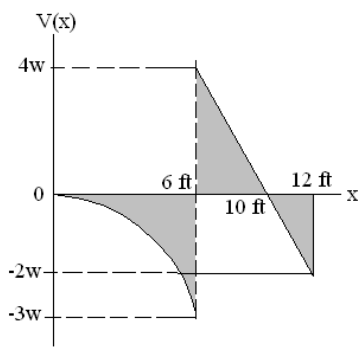
$$= -\frac{wx^2}{2} + 10wx - 48w$$

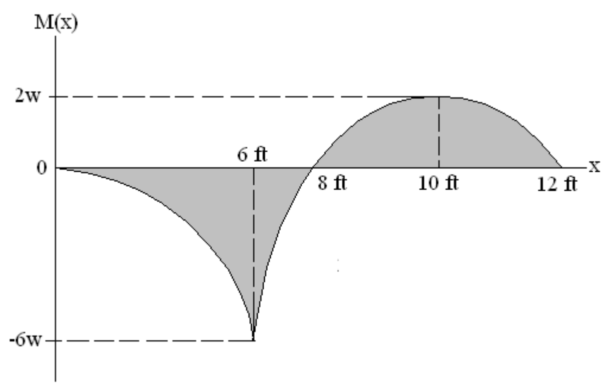
$$x = 12^- : V(12^-) = 10w - w(12) = -2w$$

$$M(12^-) = -\frac{w(12)^2}{2} + 10w(12) - 48w = 0$$

$$x = 12^+ : V(12^+) = V(12^-) + By = -2w + 2w = 0 \qquad OK$$

$$M(12^+) = M(12^-) + 0 = 0 \qquad OK$$



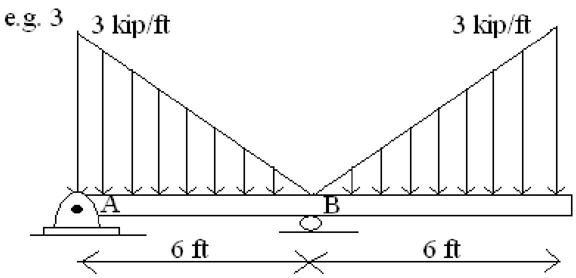


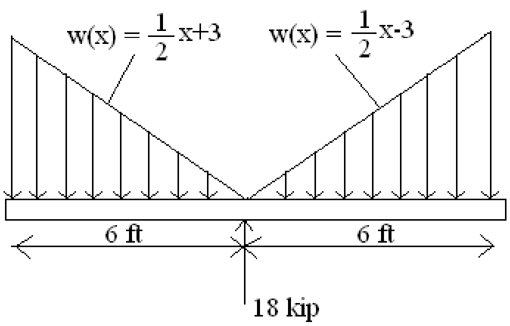
*M_{max} = 30 kip*ft = 6w* *Vmax = 8 kip = 4w*

$\Rightarrow$ *w_{max} = 5 kip/ft* $\Rightarrow$ *w_{max} = 2 kip/ft*

w_{max} = ***2 kip/ft***

e.g. 3
Given: Beam with supports shown. Symmetrical distributed loads.
Find: SFD and BMD for the beam.





Support reactions:
From symmetry, Ay = Ax = 0

$$By = 2*[\frac{1}{2}(6)(3)] = 18kip$$

$$x = 0: V(0) = 0 \quad M(0) = 0$$

$$0 \le x \le 6^- : V(x) = V(0) - \int_0^x -\frac{1}{2}x + 3dx = \frac{1}{4}x^2 - 3x$$

$$M(x) = M(0) + \int_0^x \frac{1}{4}x^2 - 3xdx = \frac{1}{12}x^3 - \frac{3}{2}x^2$$

$$x = 6^- : V(6^-) = \frac{1}{4}(6)^2 - 3(6) = -9$$

$$M(6^-) = \frac{1}{12}(6)^3 - \frac{3}{2}(6)^2 = -36$$

$$x = 6^+ : V(6^+) = V(6^-) + 18 = 9$$

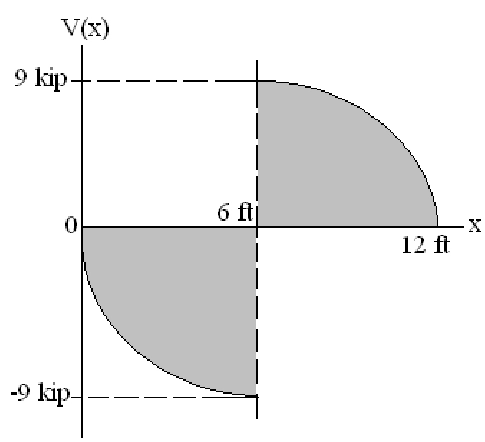
$$M(6^+) = M(6^-) + 0 = -36$$

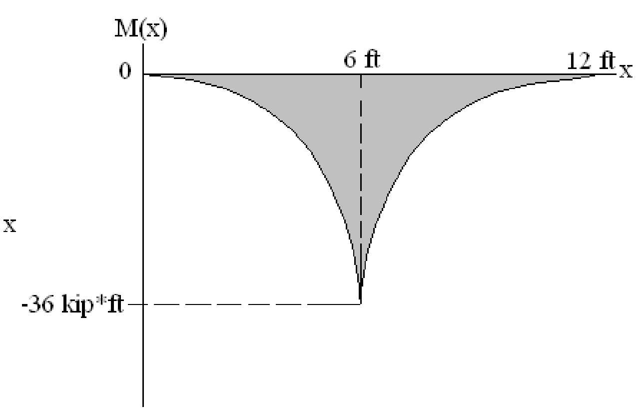
$$6^+ \le x \le 12 : V(x) = V(6^+) - \int_6^x \frac{1}{2}x - 3dx = 9 - \frac{1}{4}x^2 + 3x + 9 - 18 = 3x - \frac{1}{4}x^2$$

$$M(x) = M(6^+) + \int_6^x 3x - \frac{1}{4}x^2dx = -36 + \frac{3x^2}{2} - \frac{1}{12}x^3 - 54 + 18$$

$$= -\frac{1}{12}x^3 + \frac{3}{2}x^2 - 72$$

$$x = 12 : V(12) = 3(12) - \frac{1}{4}(12)^2 = 0 \quad OK \qquad M(12) = -\frac{1}{12}(12)^3 + \frac{3}{2}(12)^2 - 72 = 0 \quad OK$$





note: w(x) from $6^{+} \leq x \leq 12$ is $\frac{1}{2}x - 3$ NOT just $\frac{1}{2}x$ because our origin is at the far left side of the beam, not the middle of the beam.

Hibbeler, R.C. Engineering Mechanics: Statics Tenth Edition. Pearson. Upper Saddle River, NJ 2004.
Johnson, Erik. Lecturer. Univ. of Southern California. CE205. Fall 2004.