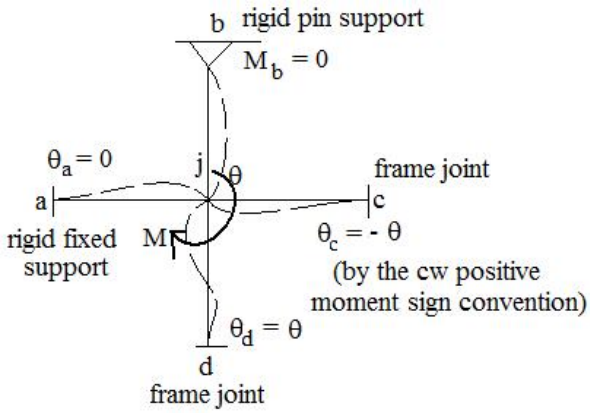
Modified stiffness method – Shortcut for certain special cases



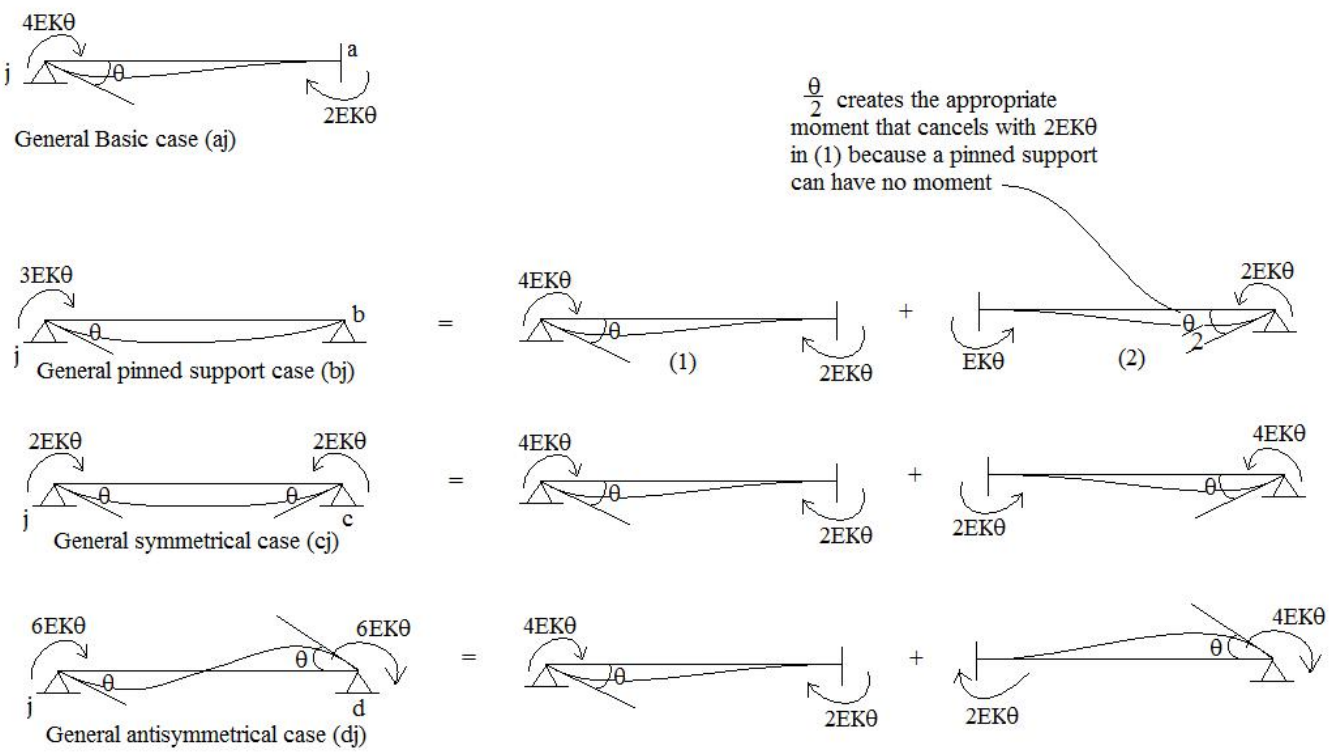
Special cases :

aj = the most basic member with a single DM at ja and COM at aj

bj = supported by a pin

cj = symmetrical

dj = antisymmetrical



Basic: $k_{ja}' = k_{ja}$, where k' = "modified stiffness factor"

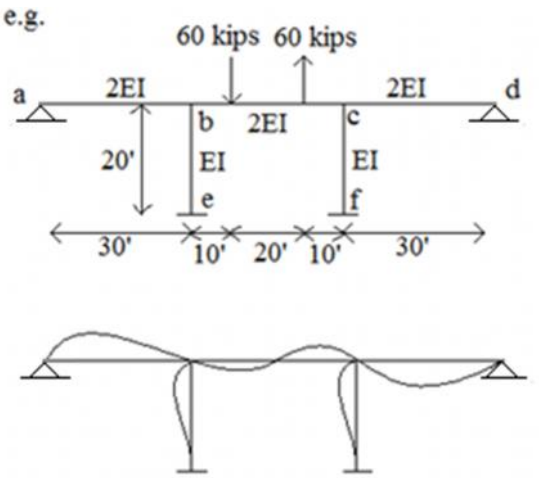
Pinned support : $M_{jb} = 3Ek_{jb}\theta = 4Ek_{jb}'\theta$, where $k_{jb}' = \frac{3}{4}k_{jb}$

Symmetrical : $M_{jc} = 2Ek_{jc}\theta = 4Ek_{jc}'\theta$, where $k_{jc}' = \frac{1}{2}k_{jc}$

Antisymmetrical : $M_{jd} = 6Ek_{jd}\theta = 4Ek_{jd}'\theta$, where $k_{jd}' = \frac{3}{2}k_{jd}$

$$M_{ja} = \frac{k_{ja}'}{\Sigma k'}M \qquad M_{jb} = \frac{k_{jb}'}{\Sigma k'}M \qquad M_{jc} = \frac{k_{jc}'}{\Sigma k'}M \qquad M_{jd} = \frac{k_{jd}'}{\Sigma k'}M$$

e.g.



$$FEM_{bc} = FEM_{cb} = (\frac{10}{40})^2(\frac{30}{40})(60)(40)$$

$$-(\frac{10}{40})(\frac{30}{40})^2(60)(40) = -225kips$$

$$k_{ba} = \frac{2I}{30} = \frac{I}{15} \quad k_{bc} = \frac{2I}{40} = \frac{I}{20} = k_{cb}$$

$$k_{cd} = \frac{2I}{30} = \frac{I}{15} \quad k_{be} = \frac{I}{20} \quad k_{cf} = \frac{I}{20}$$

modified stiffness:

$k_{ba}' = \frac{3}{4}k_{ba} = .05I = k_{cd}'$ *(pinned/roller support)*

$k_{bc}' = \frac{3}{2}k_{bc} = .075I = k_{cb}'$ *(antisymmetrical)*

$k_{be}' = k_{be} = .05I = k_{cf}'$

	ab	eb	be	ba	bc	cb	cd	cf	fc	dc
k'/Σk	1	0	.2857	.2857	.4286	.4286	.2857	.2857	0	1
FEM					-225	-225				
DM			64.3	64.3	96.4	96.4	64.3	64.3		
COM		32.15							32.15	
Σ	0	32.15	64.3	64.3	-128.6	-128.6	64.3	64.3	32.15	0

(kips * ft)

note: only one cycle needed for this problem with modified stiffness approach

note: since the frame is symmetrical, we don't really need to tabularize all of the moments, but rather just half of the frame

This problem is solved without modified stiffness in Hsieh (1995).

Works Cited

Hsieh, Yuan-Yu, and S.T. Mau. Elementary Theory of Structures: Fourth Edition. Prentice Hall. Upper Saddle River, NJ 1995.

Trifunac, Mihailo. Lecturer. University of Southern California. CE358. Fall 2005.