Simple statically indeterminate system (axial)

The following problems will be our first look at statically indeterminate (redundant) systems, as described in the section titled "A note on redundant systems" in the outline on Statics. By knowing the properties of the materials and Hooke's Law, which essentially relates force and displacement, we now have an additional equation to use for the purpose of finding unknown forces. This is called an equation of compatibility. All we have to do is find a way to relate displacements in members where we have unknown force(s).

e.g. 1

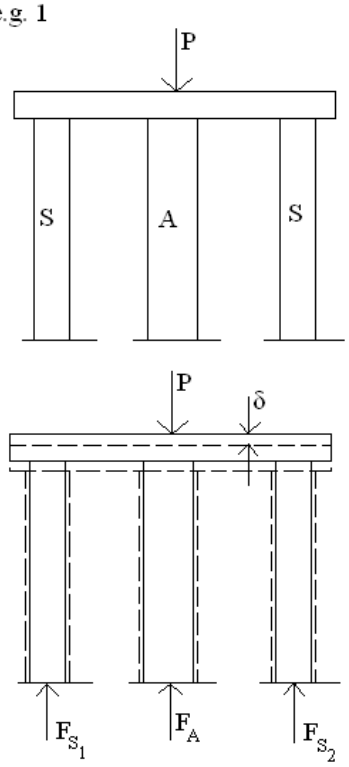
Given: Rigid bar of negligible weight rests on top of aluminum and steel beams. Force P acts at the midpoint.

Aluminum beam: diameter $d_A = 1m$, $(\sigma_A)_{allowed} = 80MPa, E_A = 70GPa$

Steel beams: diameter $d_S = .5m$, $(\sigma_S)_{allowed} = 220MPa, E_S = 210GPa$

Find: P_{max}

e.g. 1



From symmetry (or $\sum M_A = 0$*),*

$F_{S1} = F_{S2} = F_S$

$+\uparrow \sum Fy : F_A + 2F_S - P = 0$

Equation of compatibility:

Since the top bar is rigid,

$\delta_S = \delta_A$

$$\frac{F_S L}{E_S A_S} = \frac{F_A L}{E_A A_A} \quad (1)$$

From $\sum Fy = 0$, $F_A = P_{\max} - 2F_S$ *(2)*

$$(\sigma_S)_{allowed} = \frac{F_S}{A_S} \quad (3) \quad \text{and check} \quad (\sigma_A)_{allowed} \geq \frac{F_A}{A_A}$$

OR

$$(\sigma_A)_{allowed} = \frac{F_A}{A_A} \quad (3) \quad \text{and check} \quad (\sigma_S)_{allowed} \geq \frac{F_S}{A_S}$$

3 equations, 3 unknowns $F_A, F_S, P_{\max} \rightarrow$ P_{max} = ***144 MN (at which point steel yields)***

e.g. 2

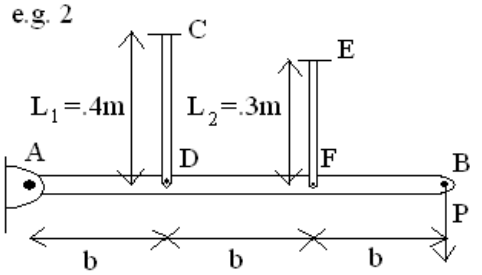
Given: Force P acts at the end of a rigid, pinned bar.

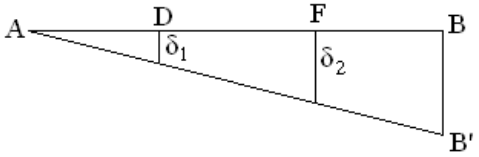
wire 1: $d_1 = 4mm, (\sigma_1)_{allowed} = 200MPa, E_1 = 72GPa$

wire 2: $d_2 = 3mm, (\sigma_2)_{allowed} = 175MPa, E_2 = 45GPa$

Find: P_{max}

e.g. 2





$+\circlearrowleft \sum M_A : T_1 b + T_2(2b) - P(3b) = 0$

$+\uparrow \sum Fy : T_1 + T_2 - Ay - P = 0$

Equation of compatibility:

From similar triangles,

$$\frac{\delta_2}{2b} = \frac{\delta_1}{b} \Rightarrow \delta_2 = 2\delta_1$$

$$\frac{T_2 L_2}{E_2 A_2} = \frac{2T_1 L_1}{E_1 A_1} \quad (1)$$

From $\sum M_A = 0,\ T_1 = 3P_{\max} - 2T_2$ *(2)*

From $\sum Fy = 0,\quad Ay = T_1 + T_2 - P$ *(3)*

$$(\sigma_1)_{allowed} = \frac{T_1}{A_1} \quad (4) \quad \text{and check: } (\sigma_2)_{allowed} \geq \frac{T_2}{A_2}$$

OR

$$(\sigma_2)_{allowed} = \frac{T_2}{A_2} \quad (4) \quad \text{and check: } (\sigma_1)_{allowed} \geq \frac{T_1}{A_1}$$

4 equations, 4 unknowns $T_1, T_2, Ay, P_{\max} \rightarrow P_{\max} =$ ***1.26kN (at which point wire 2 yields)***

Gere, James M. Mechanics of Materials: Sixth Edition. Brooks/Cole. Belmont, CA 2004.

Lee, Vincent. Lecturer. University of Southern California. CE225. Spring 2005.