Mohr's Circle

$$\sigma_{x1x1} = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{\sigma_{xx} - \sigma_{yy}}{2}\cos 2\theta_1 + \tau_{xy}\sin 2\theta_1$$

and

$$\tau_{x1y1} = \frac{-(\sigma_{xx} - \sigma_{yy})}{2}\sin 2\theta_1 + \tau_{xy}\cos 2\theta_1$$

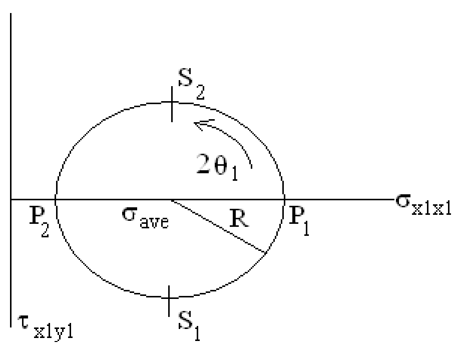
are the parametric equations of a circle.

Manipulation: Bring $\frac{\sigma_{xx} + \sigma_{yy}}{2}$ to the left side of the top equation, square both sides of the equation, and then add the two equations;

$(\sigma_{x1x1} - \sigma_{ave})^2 + \tau_{x1y1}{}^2 = R^2 \Rightarrow R = \sqrt{(\frac{\sigma_{xx} - \sigma_{yy}}{2})^2 + \tau_{xy}{}^2}$ is the algebraic equation of a circle.

Knowing $\sigma_{xx}, \sigma_{yy}, \tau_{xy}$:

- we can now find τ_{x1y1} directly from σ_{x1x1} without knowing θ (and vice-versa).
- we can construct the circle with an accurate scale and immediately see all values of τ_{x1y1} and σ_{x1x1} and their corresponding θ_1 (by measuring $2\theta_1$ with a protractor).



note: τ_{x1y1} is positive downward, and $\theta = 0°$ does NOT necessarily start at the σ_{x1x1} axis.

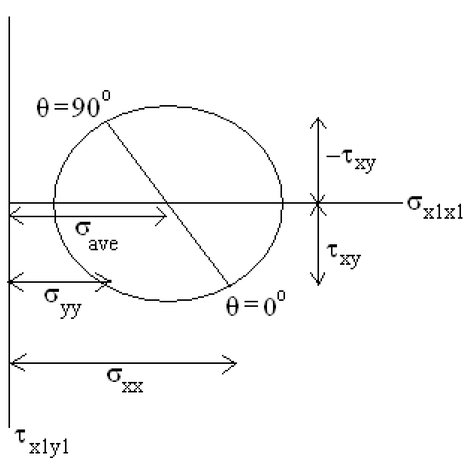
note: @ P_1 and P_2, $\tau = 0$

@ S_1 and S_2, $\sigma = \sigma_{ave}$

$\theta_{s1} = \theta_{p1} - 45°$, all as expected.

Also,

$$(\tau_{x1y1})_{max} = \text{radius} = \frac{\text{diameter}}{2} = \frac{(\sigma_{x1x1})_{max} - (\sigma_{x1x1})_{min}}{2}$$



Procedure for drawing Mohr's Circle;

First, draw axes.

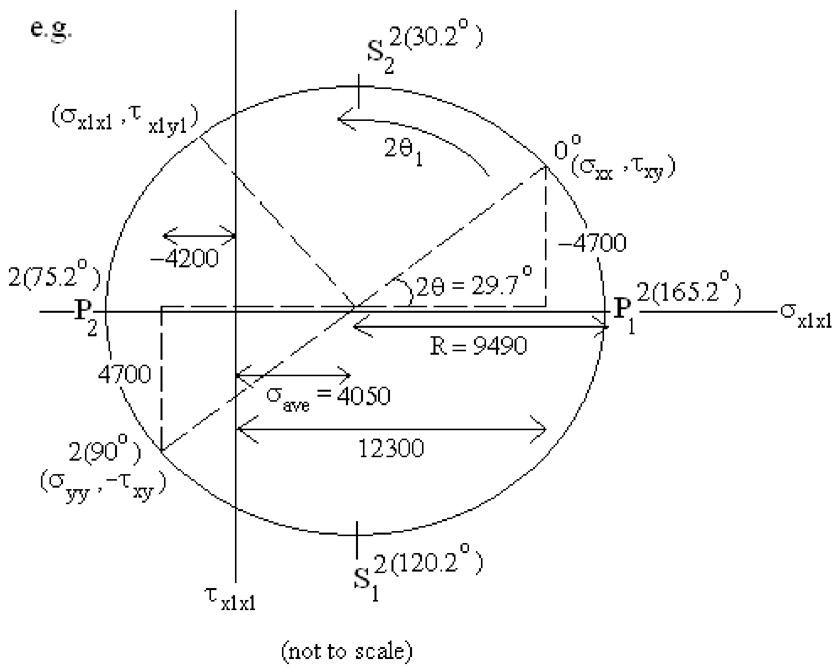
Second, the center of the circle corresponds to $(\sigma_{ave}, 0)$.

- A straight line through the center connecting (σ_{xx}, τ_{xy}) and $(\sigma_{yy}, -\tau_{xy})$ is the circle's diameter. (2R also = diameter)
- $\theta = 0$ corresponds to (σ_{xx}, τ_{xy}), $2\theta = 180$ corresponds to $(\sigma_{yy}, -\tau_{xy})$

- Draw the circle
- Find τ and σ values of interest directly from circle (if drawn with accurate scale), or from transformation equations, or using trig.

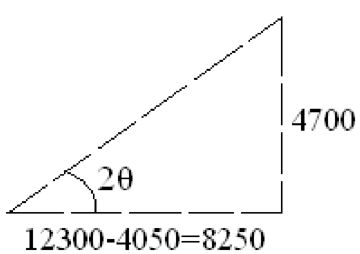
If we know the stresses σ_{x1x1}, σ_{y1y1}, and τ_{x1y1}, at a known angle θ_1, we can construct the circle first in terms of these stresses and then move <u>clockwise</u> $2\theta_1$ for σ_{xx}, σ_{yy}, and τ_{xy}.

e.g.
Given: $\sigma_{xx} = 12300\,psi \quad \sigma_{yy} = -4200\,psi \quad \tau_{xy} = -4700\,psi$
Find: $\theta_{p1}, \theta_{p2}, \theta_{s1}, \theta_{s2}, \sigma_{xp1xp1}, \sigma_{xp2xp2}, \tau_{xs1ys1}, \tau_{xs2ys2}$ *AND stresses at* $\theta_1 = 45°$



$$\sigma_{ave} = \frac{12300 + (-4200)}{2} = 4050$$

$$R = \sqrt{\left(\frac{12300 - (-4200)}{2}\right)^2 + (-4700)^2} = 9490$$



$$2\theta = \tan^{-1}\left(\frac{4700}{8250}\right) = 29.7°$$

$$2\theta_{p2} = 180 - 29.7 = 150.3°$$

$$\theta_{p2} = 75.2°$$

$\sigma_{xp2xp2} = \sigma_{ave} - R =$ ***-5440 psi***

$2\theta_{p1} = 360 - 29.7 = 330.3° \quad \theta_{p1} = 165.2°$

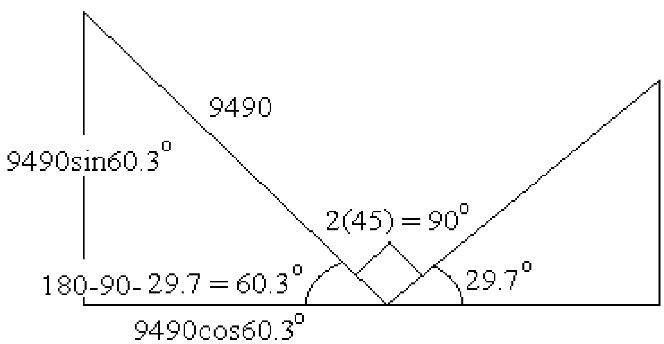
$\sigma_{xp1xp1} = \sigma_{ave} + R = \mathbf{13540\ psi}$

$2\theta_{s2} = 90 - 29.7 = 60.3° \quad \theta_{s2} = 30.2°$

$\tau_{xs2ys2} = -R = \mathbf{-9490\ psi}$

$2\theta_{s1} = 270 - 29.7 = 240.3° \quad \theta_{s1} = 120.2°$

$\tau_{xs1ys1} = R = \mathbf{9490\ psi}$



$\tau_{x1y1} = -9490 \sin 60.3° = \mathbf{-8245\ psi}$

$\sigma_{x1x1} = 4045 - 9490 \cos 60.3° = \mathbf{-660\ psi}$

- *compare solutions with the previous example*

Gere, James M. Mechanics of Materials: Sixth Edition. Brooks/Cole. Belmont, CA 2004.

Lee, Vincent. Lecturer. University of Southern California. CE225. Spring 2005.