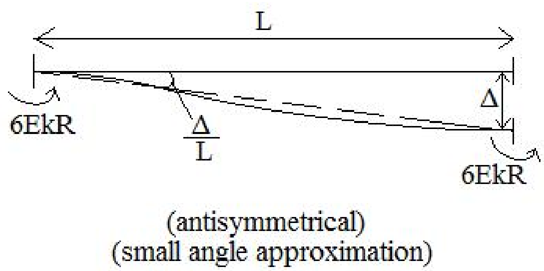
Treatment of joint translations

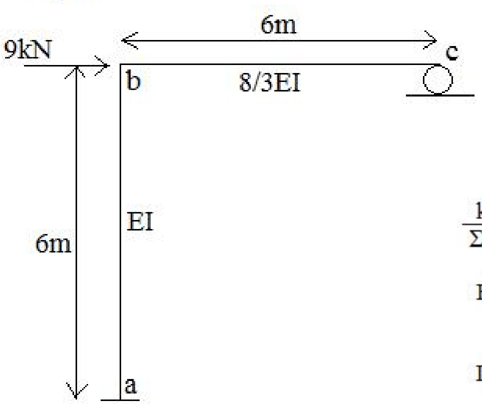


If there are joint translations, then
$M_{ja} = DM_{ja} + COM_{ja} + FEM_{ja}$
+ additional joint translation moment

relative displacement, shown, results in negative moment, shown
$R = \Delta / L$

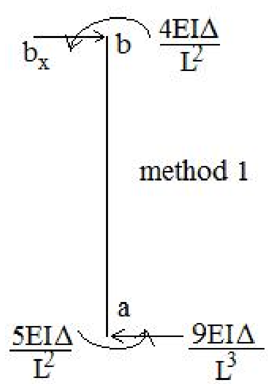
e.g. 1

e.g. 1



$$\frac{(8/3)I}{6}\left(\frac{3}{4}\right) = k_{bc}' \quad \frac{I}{6} = k_{ba}'$$

	ab	ba	bc	cb
$\frac{k'}{\Sigma k'}$	0	1/3	2/3	1
FEM	$\frac{-6EI\Delta}{L^2}$	$\frac{-6EI\Delta}{L^2}$		
DM		$\frac{2EI\Delta}{L^2}$	$\frac{4EI\Delta}{L^2}$	
COM	$\frac{EI\Delta}{L^2}$			
Σ	$\frac{-5EI\Delta}{L^2}$	$\frac{-4EI\Delta}{L^2}$	$\frac{4EI\Delta}{L^2}$	0



$\frac{9EI\text{a}}{L^3}$ *from* $\sum M_b = 0$ *for member ab*

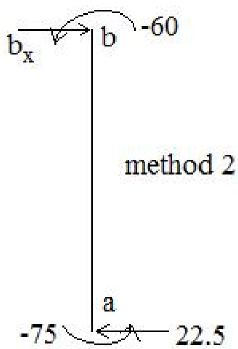
From $\sum F_x = 0$ *for the whole frame,* $\frac{9EI\text{a}}{L^3} = 9 \Rightarrow \text{a} = \frac{L^3}{EI}$

$\Rightarrow M_{ab} = -30kN*m \quad M_{ba} = -24kN*m \quad M_{bc} = 24kN*m$

OR

*Choose arbitrary FEM = -90 kN*m*

	ab	ba	bc	cb
FEM	-90	-90		
DM		30	60	
COM	15			
Σ	-75	-60	60	0



22.5 from $\sum M_b = 0$ *for member ab*

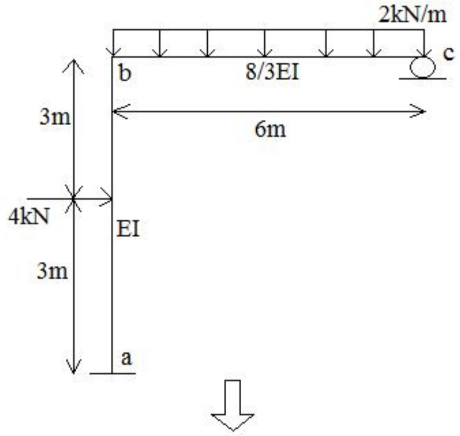
From $\sum F_x = 0$ *for the whole frame, 22.5* $\neq$ *9*

Need correction factor of $9/22.5$ *for all forces and moments*

$\Rightarrow \quad M_{ab} = -30kN*m \quad M_{ba} = -24kN*m \quad M_{bc} = 24kN*m$

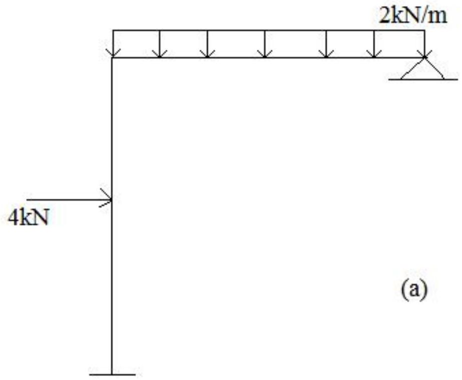
note: method 2 does not even use the formula $M = -6EkR$*. Since the solutions for method 1 and 2 match correctly, we know our derivation for* $M = -6EkR$ *is correct.*
Method 2 is more manageable, but how can we use method 2 if the loads are <u>not</u> only located at the joints?

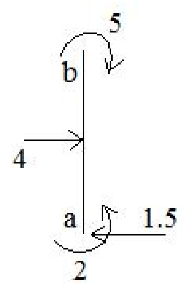
e.g. 2



(a)

	ab	ba	bc	cb
$\frac{k'}{\Sigma k'}$	0	1/3	2/3	1
FEM	-3	3	-6	6
DM		1	2	-6
COM	.5		-3	
DM		1	2	
COM	.5			
Σ	-2	5	-5	0

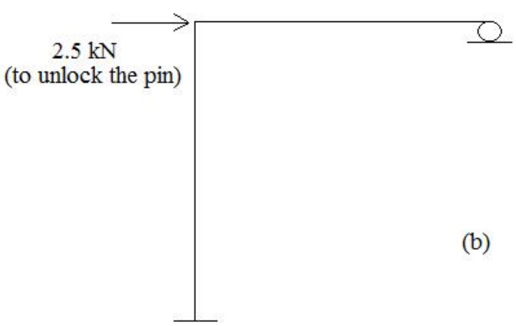




From $\sum F_x = 0$ for the whole frame, the pin has a reaction of 2.5kN acting left. This must be counter-acted as shown in (b) since the beam does <u>not</u> have a pin.

(from $\sum M_b$ for member ab)

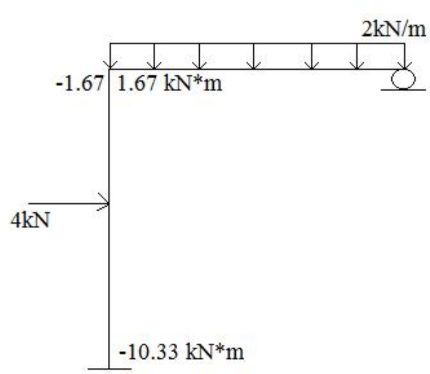
+



(b)

	ab	ba	bc	cb
FEM	-90	-90		
DM		30	60	
COM	15			
Σ	-75	-60	60	0

From $\sum M_b = 0$ for member ab and $\sum F_x = 0$ for the whole frame, the correction factor is $2.5/22.5$, where 22.5 is from the previous example.
So, moments for b:



ab	ba	bc	cb
−8.33	−6.67	6.67	0

$(a) + (b) \Rightarrow \ M_{ab} = -10.33 \quad M_{ba} = -1.67 \quad M_{bc} = 1.67\ kN * m$

note: moment distribution can be used to find moments that include the effect of sway for asymmetrical vertical loadings, and for multi-story frames, but it quickly becomes cumbersome to do by hand.

Works Cited

Hsieh, Yuan-Yu, and S.T. Mau. Elementary Theory of Structures: Fourth Edition. Prentice Hall. Upper Saddle River, NJ 1995.

Trifunac, Mihailo. Lecturer. University of Southern California. CE358. Fall 2005.