Force equilibrium

For an object not to be accelerating, the following *must be* satisfied:

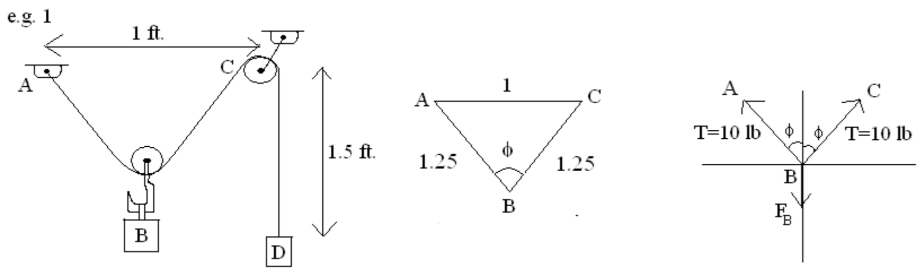
$\sum Fx = 0$

$\sum Fy = 0$

e.g. 1

Given: The total length of cord is 4 feet. $\overline{CD}$ =1.5 ft. $\overline{AC}$ =1 ft. D=10lb. Ignore mass and size of pulleys.

Find: Weight of B



$$\overline{AB} = \overline{BC} = \frac{4-1.5}{2} = 1.25 \text{ ft.}$$

When 3 sides of a triangle are known: $\cos\phi = \dfrac{1.25^2 + 1.25^2 - 1^2}{2(1.25)(1.25)} = .68$

$$\Rightarrow \phi = \cos^{-1} .68 = 47°$$

$$\theta = \frac{47°}{2} = 23.5°$$

Tension=T=10 lb

$$\xrightarrow[+]{} \sum Fx : 10\sin 23.5° - 10\sin 23.5° = 0$$

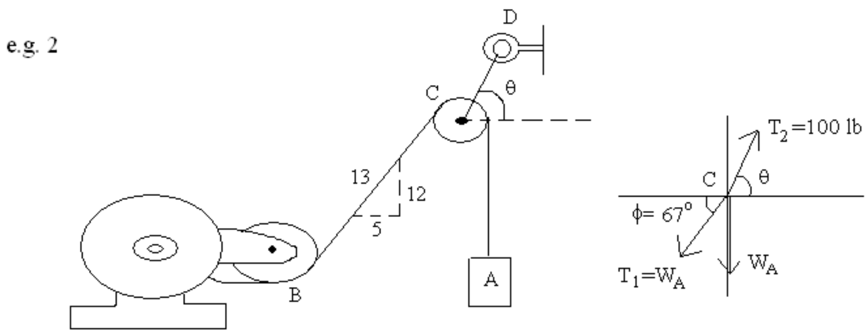
$$+\uparrow \sum Fy : 10\cos 23.5° + 10\cos 23.5° - F_B = 0 \Rightarrow F_B = \mathbf{18.3\ lb}$$

note: if F_B had turned out to be a negative value, then our assumed direction on the far right diagram would need to be reversed.

e.g. 2

Given: Crate A is to be hoisted at constant velocity ($\therefore$ this is a statics problem). Max tension in both ropes is 100 lb.

Find: θ and $(W_A)max$



$$cos\phi = \frac{5}{13} \Rightarrow \phi = 67°$$

$$\xrightarrow[+]{} \sum Fx : 100\cos\theta - T_1 \cos 67° = 0$$

$$+\uparrow \sum Fy : 100\sin\theta - T_1 \sin 67° - W_A = 0$$

But, $T_1 = W_A$

From $\sum Fx = 0$, $cos\theta = W_A cos67° \div 100 \Rightarrow \theta = \cos^{-1}(W_A cos67° \div 100)$

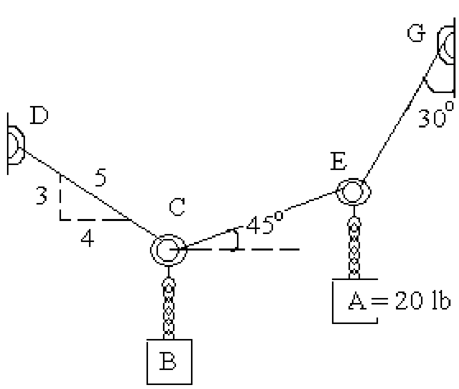
Substitute θ into $\sum Fy$:

*$100sin[\cos^{-1}(W_A cos67° \div 100)] - W_A sin67° - W_A = 0 \;\; \Rightarrow \;\; W_A =$ **51 lb***

*$\theta =$ **78.5°***

note: If W_A turned out to be > 100, then we would need to redo the calculation, this time setting W_A=100 and solving for T_2. This would then yield the correct value of θ and T_2.

e.g. 3



Draw a free body diagram at E and solve for T_{EC} and T_{EG}. Then, draw a free body diagram at C and solve for T_{CD} and W_B.

note: Resultants in three dimensional space are easily found as well, using
$$\sum Fx = \sum Fy = \sum Fz = 0$$

note: Methods that utilize equilibrium are generally preferred in engineering, whereas vector methods such as the parallelogram law and the "dot product" are typically used to illustrate math concepts.

Hibbeler, R.C. Engineering Mechanics: Statics Tenth Edition. Pearson. Upper Saddle River, NJ 2004.
Johnson, Erik. Lecturer. Univ. of Southern California. CE205. Fall 2004.