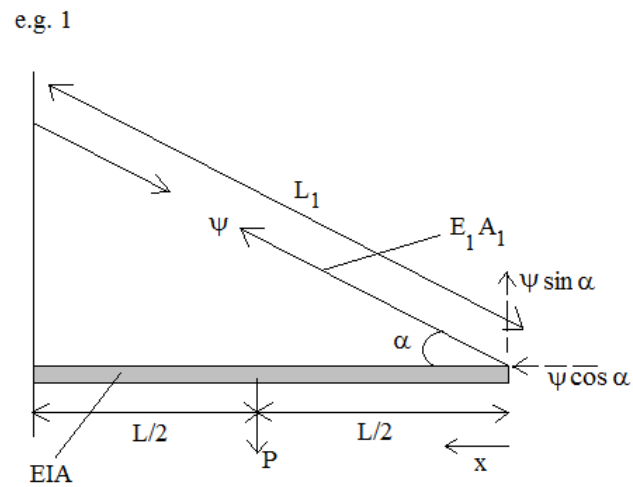
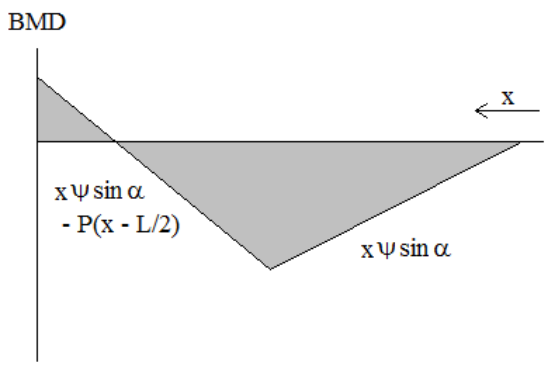
Composite Structure

e.g. 1





$$W = \int \frac{M^2 dL}{2EI} + \Sigma \frac{S^2 L_1}{2AE}$$

$$\frac{dW}{d\Psi} = 0$$

$$W = \int_0^{L/2} \frac{x^2 \Psi^2 \sin^2 \alpha}{2EI} dx$$

$$+ \int_{L/2}^{L} \frac{(x\Psi \sin\alpha - P(x - L/2)^2}{2EI} dx$$

$$+ \frac{\Psi^2 \cos^2 \alpha\, L}{2AE} + \frac{\Psi^2 L_1}{2A_1 E_1}$$

$$\frac{dW}{d\Psi} = \int_0^{L/2} \frac{x^2 \Psi \sin^2 \alpha}{EI} dx$$

$$+ \int_{L/2}^{L} \frac{[x\Psi \sin\alpha - P(x - L/2)](x \sin\alpha)}{EI} dx$$

$$+ \frac{\Psi \cos^2 \alpha\, L}{AE} + \frac{\Psi L_1}{A_1 E_1} = 0$$

$$SOLVE \Rightarrow \Psi = \frac{P \sin\alpha (\frac{5}{48}) L^3}{(\sin^2 \alpha \frac{L^3}{3} + \cos^2 \alpha \frac{IP}{A} + \frac{I L_1 E}{E_1 A_1})}$$

(axial deformation and bending)

note: If $\alpha \to 90°$ *and* $E_1 \to \infty$, *then we have a prop-cantilevered beam with* $\Psi = \frac{5P}{16}$

Choose values:

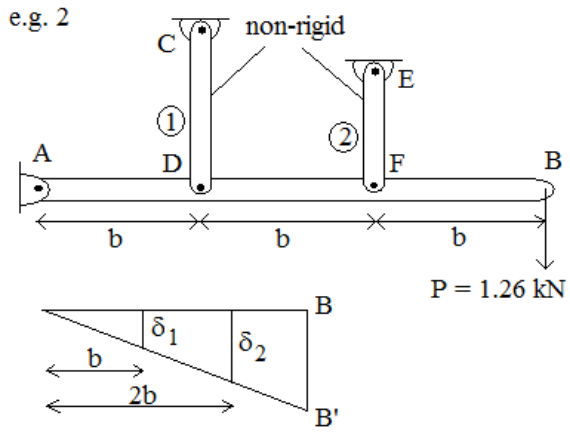
$E = E_1,\ P = 20,\ \alpha = 30°,\ L = 10,\ L_1 = 12,\ I = \frac{\pi}{64}(.3)^4,\ A = \pi(.3)^2,\ A_1 = \pi(.1)^2$

$\Rightarrow$ *(skip work)*

$\Psi = 12.47$ *(with axial deformation)*

$\Psi = 12.50$ *(without inclusion of axial deformation)*

e.g. 2



Axial Deformation Only :
DC and FE deform axially;
No bending allowed in ADFB

$E_1 = E_2$

$L_1 = .4m \quad d_1 = .004m$

$L_2 = .3m \quad d_2 = .003m$

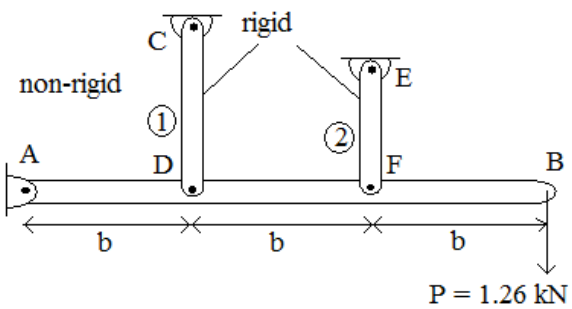
compatibility method:

$\circlearrowleft^{+} \sum M_A : T_1 b + T_2(2b) - 1.26(3b) = 0 \quad (1)$

$$\frac{\delta_2}{2b} = \frac{\delta_1}{b} \Rightarrow \delta_2 = 2\delta_1 \qquad \frac{T_2(.3)}{E(\frac{\pi}{4}(.003)^2)} = \frac{2T_1(.4)}{E(\frac{\pi}{4}(.004)^2)} \qquad (2$$

$T_2 = 1.4175kN \quad T_1 = .945kN$ *(axial deformation only)*
(different lengths and areas will result in different solutions)

Bending Only :

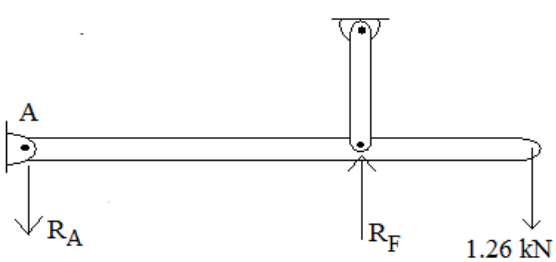


ADFB deforms in bending ; no axial deformation allowed in DC or FE

Unit load method :

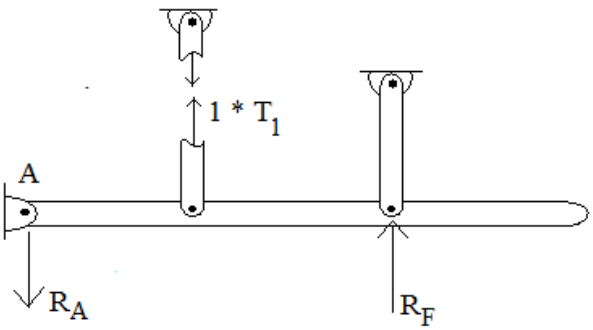
$\circlearrowleft^{+} \sum M_A : R_F(2b) - 1.26(3b) = 0 \Rightarrow R_F = 1.89kN$

$\uparrow^{+} \sum F_y : 1.89 - 1.26 - R_A = 0 \Rightarrow R_A = .63kN$



$0 \le x \le 2b^- : \quad M(x) = -.63x$

$2b^+ \le x \le 3b^- : \quad M(x) = 1.26x - 3.78b$



$$\overset{+}{\circlearrowleft}\sum M_A : 1(b) - R_F(2b) = 0 \Rightarrow R_F = .5kN$$
$$\uparrow^+ \sum F_y : 1 - 1/2 - R_A = 0 \Rightarrow R_A = .5kN$$

$$0 \le x \le b^- : M(x) = -.5x$$
$$b^+ \le x \le 2b^- : M(x) = .5x - 6$$
$$2b^+ \le x \le 3b : M(x) = 0$$

$$\Delta = \int_0^b \frac{(-.63x)(-.5x)}{EI}dx + \int_b^{2b} \frac{(-.63x)(.5x-b)}{EI}dx + \int_{2b}^{3b} \frac{(1.26x - 3.78b)(0)}{EI}dx = \frac{.315b^3}{EI}$$
$$\delta = \int_0^b \frac{(-.5x)^2}{EI}dx + \int_b^{2b} \frac{(.5x-b)^2}{EI}dx + 0 = \frac{.1667b^3}{EI}$$
$$\Delta + \delta T_1 = 0 \qquad \frac{.315b^3}{EI} + \frac{.1667b^3}{EI}T_1 = 0$$

$T_1 = -1.890kN \quad T_2 = 2.835kN$ *(from equilibrium or from same process on* T_2*)*
(bending only)

To treat the structure as a composite member, we need the moment of inertia I for beam ADFB, as well as "b" :

Take $I = \frac{\pi}{64}(.006)^4 m^4, b = .3m$
Choose T_1 *as the redundant:*

Total elongation of member DC $= \left(\frac{.315b^3}{EI} + \frac{.16667b^3T_1}{EI}\right) + \frac{2T_1(.4)}{E(\frac{\pi}{4}(.004)^2)} = 0$

$\Rightarrow T_1 = -1.888kN$

The inclusion of axial deformation has no <u>*significant*</u> *effect on the solution, for this problem.*

note: Previous analysis of the statically indeterminate one bay frame (with the beam loaded uniformly) resulted in each vertical support reaction being equal to exactly one-half of the total load (obviously the exact answer) despite the neglecting of axial deformation. This is because this frame was symmetrical (there are no relative axial displacements).

Works Cited

Hsieh, Yuan-Yu, and S.T. Mau. Elementary Theory of Structures: Fourth Edition. Prentice Hall. Upper Saddle River, NJ 1995.

Trifunac, Mihailo. Lecturer. University of Southern California. CE358. Fall 2005.